

Nearly perfect sequences with arbitrary out-of-phase autocorrelation

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Abstract. In this paper we study nearly perfect sequences (NPS) via their connection to direct product difference sets (DPDS). We prove the connection between a p -ary NPS of period n and type γ and a cyclic $(n, p, n, \frac{n-\gamma}{p} + \gamma, 0, \frac{n-\gamma}{p})$ -DPDS for an arbitrary integer γ . Next, we present the necessary conditions for the existence of a p -ary NPS of type γ . We apply this result for excluding the existence of some p -ary NPS of period n and type γ for $n \leq 100$ and $|\gamma| \leq 2$. We also prove the similar results for an almost p -ary NPS of type γ . Finally, we show the non-existence of some almost p -ary perfect sequences by showing the non-existence of equivalent cyclic relative difference sets by using the notion of multipliers.

Keywords: perfect sequence, nearly perfect sequence, direct product difference set, relative difference set

1 Introduction

Let $\underline{a} = (a_0, a_1, \dots, a_{n-1}, \dots)$ be a sequence of period n with entries $a_i \in \mathbb{C}$. For $0 \leq t \leq n-1$, the *autocorrelation function* $C_{\underline{a}}(t)$ is defined by

$$C_{\underline{a}}(t) = \sum_{i=0}^{n-1} a_i \overline{a_{i+t}},$$

where \overline{a} is the complex conjugate of a . The values $C_{\underline{a}}(t)$ at $1 \leq t \leq n-1$ are called *the out-of-phase autocorrelation coefficients* of \underline{a} .

Let p be a prime and $\zeta_p \in \mathbb{C}$ be a primitive p -th root of unity. If $a_i = \zeta_p^{b_i}$ for some integer b_i , with $i = 0, 1, \dots, n-1$, then \underline{a} is called a *p -ary sequence*. If $a_0 = 0$ and all other entries are a power of ζ_p , then \underline{a} is called an *almost p -ary sequence*.

An (almost) p -ary sequence \underline{a} of period n is called *perfect sequence* (PS) if all out-of-phase autocorrelation coefficients of \underline{a} are 0. Similarly, an (almost) p -ary sequence \underline{a} of period n is called a *nearly perfect sequence* (NPS) of type γ if all out-of-phase autocorrelation coefficients of \underline{a} are a constant γ . We write a NPS of type $\gamma = 0$ to denote a PS. (We also note that there is another notion of *almost perfect sequences* which is a p -ary sequence \underline{a} of period n having $C_{\underline{a}}(t) = 0$ for all $1 \leq t \leq n-1$ -with exactly one exception. In this paper, we will study p -ary NPS and almost p -ary NPS.)

There are some applications of (almost) p -ary sequences of period n if $C_{\underline{a}}(t)$ is small for $1 \leq t \leq n-1$. We refer to [1] and the references therein for such applications.

Nearly perfect sequences have been studied widely by many authors. For instance, Jungnickel and Pott [5] studied binary nearly perfect sequences of type $|\gamma| \leq 2$, and gave existence and non-existence cases. Ma and Ng [7] studied p -ary nearly perfect sequences of type $|\gamma| \leq 1$, and determined their existence status by using their connection to direct product difference sets (DPDS). Later Chee et al. [3] extended the methods due to Ma and Ng [7] to almost p -ary nearly perfect sequences of types $\gamma = 0$ and $\gamma = -1$. Then, Özbudak et al. [9] proved the non-existence of almost

p -ary perfect sequences at certain values. Recently, Winterhof et al. [12] studied the existence of (almost) m -ary NPS for an integer m via their connection to Butson-Hadamard matrices, certain Diophantine equations and ideal decomposition.

In this paper we prove a general equality between an (almost) p -ary (nearly) perfect sequence of type γ for an arbitrary $\gamma \in \mathbb{Z}$ and a direct product difference set. Firstly, we prove the connection between a p -ary NPS of type γ and a DPDS for an arbitrary integer γ (see Theorem 1). By this result we prove necessary conditions for the existence of a p -ary NPS of type γ (see Theorem 2). Then we demonstrate the pairs (n, p) such that the existence of a p -ary NPS of period n and type γ is excluded by Theorem 2 for $n \leq 100$ and $\gamma = -1, 0, 1, 2$. In particular, we exclude the existence of a 23-ary NPS of period 45 and type $\gamma = -1$ by Theorem 2, which was an undecided case in [7]. We note that an (almost) p -ary NPS of period n and type γ does not exist if $n \geq 3$ and $\gamma \leq -2$ (see Lemma 3 below).

Next, we prove the counterpart of the results on a p -ary NPS of type γ for an almost p -ary NPS of type γ where $\gamma \in \mathbb{Z}$ by considering its connection to a DPDS (see Theorems 3 and 4). And, we demonstrate the pairs (n, p) such that the existence of an almost p -ary NPS of period $n + 1$ and type γ are excluded by Theorem 4 for $n \leq 100$ and $\gamma = -1, 0, 1, 2$. In particular, we exclude the existence of a 7-ary NPS of period 77 and type $\gamma = -1$ by Theorem 4, which was an undecided case in [3]. Furthermore, we present a generalization of these results for showing the non-existence of an almost p -ary NPS with $s \geq 1$ zero-symbols.

Finally, we show the non-existence of certain almost p -ary PS of period n via showing the non-existence of a regular $(n + 1, p, n, \frac{n-1}{p})$ -RDS by using the notion of multipliers (see Theorem 5).

The paper is organized as follows. In Section 2, we present the definition of DPDS and preliminary results that we use later. We present our result on p -ary NPS of type γ in Section 3, and on almost p -ary NPS of type γ in Section 4. Finally, we present a result by using the notion of multipliers in Section 5.

2 Preliminaries

We begin with the definition of a direct product difference set [7].

Definition 1. Let $G = H \times N$, where the order of H and N are m and n . A subset R of G is called an $(m, n, k, \lambda_1, \lambda_2, \mu)$ direct product difference set (DPDS) in G relative to H and N if both of the following statements hold:

- (i.) $|R| = k$,
- (ii.) Differences $r_1 r_2^{-1}$, $r_1, r_2 \in R$ with $r_1 \neq r_2$ represent
 - all non identity elements of H exactly λ_1 times,
 - all non identity elements of N exactly λ_2 times,
 - all non identity elements of $G \setminus H \cup N$ exactly μ times.

We can also define a difference set by using the group-ring algebra notation. Let $\sum_{g \in R} g \in \mathbb{Z}[G]$ be an element of the group ring $\mathbb{Z}[G]$, for simplicity we will denote the sum by R . If R is an $(m, n, k, \lambda_1, \lambda_2, \mu)$ -DPDS in G relative to H and N then

$$RR^{(-1)} = (k - \lambda_1 - \lambda_2 + \mu) + (\lambda_1 - \mu)H + (\lambda_2 - \mu)P + \mu G \quad (1)$$

holds in $\mathbb{Z}[G]$.

We now present two known results that we will use in subsequent sections. The following lemma is noticed by Turyn [11]. Let q be a prime and $u = q^r w$ where $\gcd(q, w) = 1$. We say that q is self-conjugate modulo u if $q^j \equiv -1 \pmod{w}$ for some integer j .

Lemma 1. If q is self conjugate modulo u , then $\overline{Q} = Q$ for any prime ideal divisor Q of $q\mathbb{Z}[\zeta_u]$.

Next result is known as Ma's Lemma [7]. We denote by L^\perp the subset of the character group which is principal on L .

Lemma 2. Let q be a prime and α be a positive integer. Let K be an abelian group such that either q does not divide $|K|$ or the Sylow q -subgroup of K is cyclic. Let L be any subgroup of K and $Y \in \mathbb{Z}[K]$ where coefficients of Y lie between a and b where $a < b$. Suppose that

1. q is self conjugate modulo $\exp(K)$,
2. $q^r \mid \chi(Y)\overline{\chi(Y)}$ for all $\chi \notin L^\perp$ and $q^{r+1} \nmid \chi(Y)\overline{\chi(Y)}$ for some $\chi \notin L^\perp$,
3. $\chi(Y) \neq 0$ for some $\chi \notin L^\perp \cup Q^\perp$ where $Q = K$ if $q \nmid |K|$, and Q is the subgroup of K of order q otherwise.

Then

1. if $q \nmid |K|$, r is even and $q^{r/2} \leq b - a$,
2. if Sylow q -subgroup of K is cyclic, $q^{\lfloor \frac{r}{2} \rfloor} \leq 2(b - a)$ when L is a proper subgroup of $|K|$ and $q^{\lfloor \frac{r}{2} \rfloor} \leq b - a$ when $L = K$.

In the last part of this section we give a known result on the non-existence of NPS of type γ for $\gamma \leq -2$. This is a direct consequence of [2, Theorem 2.5], see also [12, Corollary 3.1]. We give a short proof below.

Lemma 3. Let p be a prime number, $n \in \mathbb{Z}^+$ and $\gamma \in \mathbb{Z}$ such that $n \geq 2$ and $\gamma \leq 2$. Then a p -ary (almost) NPS of period n and type γ does not exist except the existence of a binary NPS of period 2 and type -2.

Proof. Assume the existence of a p -ary NPS of period $n \geq 2$ and type $\gamma \leq -2$, say $\underline{a} = (a_0, a_1, \dots, a_{n-1})$. Let $H = (h_{i,j})$ be a circulant matrix, that is $h_{i+1,j+1} = h_{i,j}$ for all i, j , defined by $h_{0,j} = a_j$ for $j = 0, 1, \dots, n-1$ then H is a circulant near Butson-Hadamard matrix of order n satisfying $H\overline{H}^T = (n - \gamma)I + \gamma J$, where I is the identity matrix and J is the all 1 square matrix of order n . Hence, $\det(H\overline{H}^T) = ((\gamma + 1)n - \gamma)(n - \gamma)^{n-1}$. Since $\det(H\overline{H}^T)$ is a non-negative number, we obtain a contradiction. We finally note that $(-1, 1)$ is a binary NPS of period 2 and type -2. The proof of the non-existence of an almost p -ary NPS of period $n \geq 2$ and type $\gamma \leq -2$ is very similar. \square

3 p -ary (nearly) perfect sequences

In the following result we prove the general connection between a p -ary sequence of type γ and a DPDS for an integer γ . This is a generalization of [7, Theorems 4.2 and 5.1]) to an arbitrary integer γ .

Theorem 1. Let p be a prime, $n \geq 2$ be an integer, and $\underline{a} = (a_0, a_1, \dots, a_{n-1}, \dots)$ be a p -ary sequence of period n . Let $H = \langle h \rangle$ and $P = \langle g \rangle$ be the (multiplicatively written) cyclic groups of order n and p , respectively. Let G be the group defined as $G = H \times P$. We choose a primitive p -th root of 1, $\zeta_p \in \mathbb{C}$. For $0 \leq i \leq n-1$ let b_i be the integer in $\{0, 1, 2, \dots, p-1\}$ such that $a_i = \zeta_p^{b_i}$. Let R be the subset of G defined as

$$R = \{(g^{b_i} h^i) \in G : 0 \leq i \leq n-1\}.$$

Then \underline{a} is a p -ary NPS of type γ if and only if R is an $(n, p, n, \frac{n-\gamma}{p} + \gamma, 0, \frac{n-\gamma}{p})$ -DPDS in G relative to H and N . In particular, p divides $n - \gamma$.

Proof. Let $A = \sum_{i=0}^{n-1} a_i h^i \in \mathbb{C}[H]$. Then we have

$$A\overline{A}^{(-1)} = \sum_{t=0}^{n-1} C_a(t) h^t.$$

Let χ be a character on P . We extend χ to G such that $\chi(h) = h$. Let $\sigma \in \text{Gal}(\mathbb{Q}(\zeta_p) \setminus \mathbb{Q})$ such that $\sigma(\zeta_p) = \chi(\zeta_p)$. If χ is a nonprincipal character on P , then we have $\chi(R) = A^\sigma$, and so

$$\chi(RR^{(-1)}) = (A\overline{A}^{(-1)})^\sigma.$$

On the other hand, if χ is a principal character on P , then we have

$$\chi(R) = H.$$

Then

$$\chi(RR^{(-1)}) = \begin{cases} nH & \text{if } \chi \text{ is principal on } P \\ \sum_{t=0}^{n-1} C_a(t)^\sigma h^t & \text{if } \chi \text{ is nonprincipal on } P \end{cases}$$

If a is a NPS of type γ , then

$$\chi(RR^{(-1)}) = \begin{cases} nH & \text{if } \chi \text{ is principal on } P \\ n - \gamma + \gamma H & \text{if } \chi \text{ is nonprincipal on } P \end{cases}$$

By extending χ to H we obtain

$$\chi(RR^{(-1)}) = \begin{cases} n^2 & \text{if } \chi \text{ is principal on } P \text{ and } H \\ 0 & \text{if } \chi \text{ is principal on } P \text{ and nonprincipal on } H \\ (\gamma + 1)n - \gamma & \text{if } \chi \text{ is nonprincipal on } P \text{ and principal on } H \\ n - \gamma & \text{if } \chi \text{ is nonprincipal on } P \text{ and nonprincipal on } H \end{cases}$$

On the other hand it is easy to see by using (1) that the same diagram holds for an $(n, p, n, \frac{n-\gamma}{p} + \gamma, 0, \frac{n-\gamma}{p})$ -DPDS for any character χ on G . Therefore we are done. \square

Theorem 1 gives a way of showing the existence and the non-existence of NPS via using DPDS. We will use this method in proving the following theorem. For integers q, r and n we use $q^r || n$ to denote that $q^r | n$ but $q^{r+1} \nmid n$.

Theorem 2. *Let p be prime number, $n \in \mathbb{Z}^+$ and $\gamma \in \mathbb{Z}$ such that $|\gamma| < n$ and $p | n - \gamma$. Suppose that there exists a p -ary NPS of type γ and period n .*

- (i) *For $\gamma = 0$, let $q \neq p$ be prime number dividing n such that $q^r || n$ and q is self-conjugate modulo up for some divisor $u \geq 1$ of n . Then r is even and $q^{r/2} \leq \frac{n}{u}$.*
- (ii) *For $\gamma \neq 0$, let $q \neq p$ be prime number dividing $n - \gamma$ such that $q^r || n - \gamma$ and q is self-conjugate modulo up for some divisor $u > 1$ of n . If $q \nmid u$, then r is even and $q^{r/2} \leq \frac{n}{u}$. If $q | u$ then $q^{\lfloor r/2 \rfloor} \leq 2\frac{n}{u}$.*
- (iii) *For $\gamma \neq 0$, if $p^r || n - \gamma$ and p is self-conjugate modulo u for some divisor $u > 1$ of n such that $p \nmid u$, then $p^{r/2} \leq 2\frac{n}{u}$ in case r is even, and $p^{(r+1)/2} \leq 4\frac{n}{u}$ in case r is odd.*
- (iv) *For $\gamma \neq 0$, let $q \neq p$ be prime number dividing $(\gamma + 1)n - \gamma$ such that $q^r || (\gamma + 1)n - \gamma$ and q is self-conjugate modulo up for some divisor $u \geq 1$ of n . If $q \nmid u$, then r is even.*

Proof. We note that (i) is already proved in [7, Theorem 4.11]. Therefore, it is enough to prove (ii), (iii) and (iv). By using Theorem 1, the existence of a p -ary NPS of period n and type γ implies that there is an $(n, p, k, \frac{n-\gamma}{p} + \gamma, 0, \frac{n-\gamma}{p})$ -DPDS in $G = H \times P$ relative to $H = \langle h \rangle$ and $P = \langle g \rangle$ where $o(h) = n$ and $o(g) = p$.

Let $\rho : G \rightarrow K := G/\langle h^u \rangle$ be the natural epimorphism. By using (1) with $(n, p, k, \frac{n-\gamma}{p} + \gamma, 0, \frac{n-\gamma}{p})$ -DPDS, we have

$$\rho(RR^{(-1)}) = (n - \gamma) + \gamma \frac{n}{u} \rho(H) - \left(\frac{n - \gamma}{p}\right) \rho(P) + \left(\frac{n - \gamma}{p}\right) \frac{n}{u} K$$

The coefficients of $\rho(R)$ lie between 0 and $\frac{n}{u}$. Let χ be a nonprincipal character of K . Then

$$\chi(\rho(RR^{(-1)})) = \begin{cases} 0 & \text{if } \chi \text{ is principal on } \rho(P) \text{ and nonprincipal on } \rho(H) \\ (\gamma + 1)n - \gamma & \text{if } \chi \text{ is nonprincipal on } \rho(P) \text{ and principal on } \rho(H) \\ n - \gamma & \text{if } \chi \text{ is nonprincipal on } \rho(P) \text{ and nonprincipal on } \rho(H) \end{cases} \quad (2)$$

Assume that $q^r \parallel n - \gamma$ and q is self-conjugate modulo $\exp(K) = up$ for some divisor $u > 1$ of n . We will use Lemma 2 with $L = \rho(H)$ and $Y = \rho(R)$ to prove (ii). We note that $\rho(H)$ is nontrivial as $u > 1$. Then we have $q^r \mid \chi(Y)\overline{\chi(Y)}$ for all $\chi \notin L^\perp$ and $q^{r+1} \nmid \chi(Y)\overline{\chi(Y)}$ for $\chi \notin L^\perp \cup \rho(P)^\perp$. If $q \mid |K|$, then it is clear that $\chi(Y) \neq 0$ for some $\chi \notin \rho(Q)^\perp \cup \rho(P)^\perp$, where Q is the Sylow q -subgroup of K since $\chi(Y)\overline{\chi(Y)} = n - \gamma$. Then by Lemma 2 we get (ii).

Similarly, if $q = p$ and $p \nmid u$ then the Sylow p -subgroup of K is cyclic. When r is even, we obtain the first part of (iii) by using Lemma 2. When r is odd, we define

$$R' := R \sum_{t=1}^{p-1} \left(\frac{t}{p} \right) g^t$$

where $\left(\frac{t}{p} \right)$ denotes the Legendre symbol. We note that the coefficients of R' are $-1, 0, +1$, so the coefficients of $\rho(R')$ are between $-n/u$ and n/u . We also note that

$$\sum_{t=1}^{p-1} \left(\frac{t}{p} \right) g^t \overline{\sum_{t=1}^{p-1} \left(\frac{t}{p} \right) g^t} = p$$

by using [8, Lemma 4.5]. And so, $p^{r+1} \mid \chi(\rho(R'))\overline{\chi(\rho(R'))}$ for all $\chi \notin L^\perp$ and $p^{r+2} \nmid \chi(\rho(R'))\overline{\chi(\rho(R'))}$ for $\chi \notin L^\perp \cup \rho(P)^\perp$. Thus by using Lemma 2 we prove the second part of (iii).

Finally, it is clear from (2) that there exists a nonprincipal character χ of K such that $\chi(Y)\overline{\chi(Y)} = (\gamma + 1)n - \gamma$. If there exists a prime number $q \neq p$ such that $q^r \parallel (\gamma + 1)n - \gamma$ and q is self-conjugate modulo up for some divisor $u \geq 1$ of n , then r must be even by Lemma 1. This proves (iv). \square

Ma and Ng [7] present tables showing the existence status of p -ary (nearly) perfect sequences of period n for $|\gamma| \leq 1$ and $2 \leq n \leq 50$. We extend the tables given in [7] to $|\gamma| \leq 2$ and $2 \leq n \leq 100$. In addition, we update the tables for some undecided cases. Below, we explain existence, non-existence and undecided cases. We present the detailed tables in Appendix. The empty rows in the tables are undecided cases. The case $p = 2$ is extensively studied in [5], therefore in this section we only deal with the case that p is an odd prime.

For $\gamma = 0$, it is known that an $(n, p, n, n/p, 0, n/p)$ -DPDS in $\mathbb{Z}_n \times \mathbb{Z}_p$ relative to \mathbb{Z}_n and \mathbb{Z}_p exists for $n = p$ and $n = p^2$ where p is an odd prime (see [10, Theorem 2.2.9] and [6, Theorem 2.3] respectively). Therefore, a p -ary PS of period n for $n = p$ and $n = p^2$ exist. For $n \leq 100$, Theorem 2 excludes the existence at all other pairs (n, p) except a few undecided cases $(n, p) \in \{(28, 7), (33, 11), (39, 13), (55, 11), (56, 7), (63, 3), (69, 23), (84, 3), (92, 23), (95, 19), (99, 11)\}$.

For $\gamma = -1$, it is known that an $(n, p, n, (n+1-p)/p, 0, (n+1)/p)$ -DPDS in $\mathbb{Z}_n \times \mathbb{Z}_p$ relative to \mathbb{Z}_n and \mathbb{Z}_p exists for $n = q - 1$ where q is a power of p (see [4]). Therefore, a p -ary NPS of period $q - 1$ for q is a power of p exists. For $n \leq 100$, Theorem 2 excludes the existence at all other pairs (n, p) except $(n, p) \in \{(19, 5), (23, 3), (27, 7), (32, 11), (35, 3), (38, 3), (41, 3), (44, 5), (47, 3), (55, 7), (56, 3), (59, 5), (65, 3), (65, 11), (67, 17), (71, 3), (73, 37), (74, 3), (76, 7), (79, 5), (83, 7), (91, 23), (92, 3), (93, 47), (98, 11), (99, 5)\}$. We note that 23-ary NPS of period 45 for $\gamma = -1$ was an undecided case in [7]. Theorem 2 (iii) with $q = 23$ and $u = 9$ shows that such a sequence does not exist. It is assumed in [7] that Theorem 2 (ii) holds for $\gamma = -1$ and $u = 1$. However, we show in the proof that it does not hold for $u = 1$. Because of this reason, we say that the cases $(23, 3)$ and $(41, 3)$ are undecided which were given to be nonexistent in [7].

For $\gamma = 1$, Theorem 2 excludes the existence of a p -ary NPS of period n for the pairs (n, p) such that $n \leq 100$ except the ones $(n, p) \in \{(5, 3), (13, 3), (15, 7), (22, 3), (25, 3), (27, 13), (31, 3), (31, 5), (40, 3), (40, 13), (45, 11), (49, 3), (51, 5), (56, 11), (57, 7), (63, 31), (64, 3), (64, 7), (70, 23), (76, 3), (79, 3), (79, 13), (85, 3), (85, 7), (95, 47), (96, 19), (97, 3), (99, 7), (100, 11)\}$. And, it is known that a p -ary NPS for $\gamma = 1$ of period n exists for pairs $(n, p) \in \{(5, 3), (13, 3)\}$ (see [7]).

For $\gamma = 2$, Theorem 2 excludes the existence of a p -ary NPS of period n for the pairs (n, p) such that $n \leq 100$ except the ones $(n, p) \in \{(5, 3), (9, 7), (11, 3), (16, 7), (17, 3), (22, 5), (23, 3), (26, 3), (30, 7), (33, 31), (35, 3), (35, 11), (37, 7), (41, 3), (50, 3), (57, 5), (58, 7), (59, 3), (59, 19), (65, 3),$

$(77,3), (81,79), (86,3), (95,3), (98,3)\}$. On the other hand, an exhaustive search says that no p -ary NPS of period n and type $\gamma = 2$ exists for the pairs $(n,p) = (9,7)$, but it exists for the pairs $(n,p) \in \{(5,3), (17,3)\}$: $(\zeta_3^2, \zeta_3^2, \zeta_3^2, \zeta_3^2, 1)$ is a 3-ary NPS of period 5 and type 2, and $(\zeta_3^2, \zeta_3^2, \zeta_3^2, 1, \zeta_3^2, 1, 1, \zeta_3, 1, 1, \zeta_3^2, 1, \zeta_3^2, \zeta_3^2, \zeta_3^2, 1, 1)$ is a 3-ary NPS of period 17 and type 2.

4 Almost p -ary (nearly) perfect sequences

In the following we prove the equality between an almost p -ary sequence of type γ and a DPDS for an integer γ . This is a generalization of [3, Theorems 1 and 6] to an arbitrary integer γ . Its proof is similar to the proof of Theorem 1.

Theorem 3. *Let p be a prime, $n \geq 2$ be an integer, and $\underline{a} = (a_0, a_1, \dots, a_n, \dots)$ be an almost p -ary sequence of period $n+1$. Let $H = \langle h \rangle$ and $P = \langle g \rangle$ be the (multiplicatively written) cyclic groups of order $n+1$ and p . Let G be the group defined as $G = H \times P$. We choose a primitive p -th root of 1, $\zeta_p \in \mathbb{C}$. For $1 \leq i \leq n$ let b_i be the integer in $\{0, 1, 2, \dots, p-1\}$ such that $a_i = \zeta_p^{b_i}$. We define $a_0 := 0$. Let R be the subset of G defined as*

$$R = \{(g^{b_i} h^i) \in G : 1 \leq i \leq n\}.$$

Then \underline{a} is an almost p -ary NPS of type γ if and only if R is an $(n+1, p, n, \frac{n-\gamma-1}{p} + \gamma, 0, \frac{n-\gamma-1}{p})$ -DPDS in G relative to H and P . In particular, p divides $n - \gamma - 1$.

Proof. We define

$$A := \sum_{i=0}^n a_i h^i \in \mathbb{C}[H],$$

where $a_0 = 0$. By applying steps in the proof of Theorem 2 we complete the proof. \square

Let χ be a character of $G = H \times P$ where G is defined as in Theorem 3. If $R \in \mathbb{Z}[G]$ is an $(n+1, p, n, \frac{n-\gamma-1}{p} + \gamma, 0, \frac{n-\gamma-1}{p})$ -DPDS in G relative to H and P , then we have

$$\chi(RR^{(-1)}) = \begin{cases} n^2 & \text{if } \chi \text{ is principal on } P \text{ and } H \\ 1 & \text{if } \chi \text{ is principal on } P \text{ and nonprincipal on } H \\ (\gamma+1)n & \text{if } \chi \text{ is nonprincipal on } P \text{ and principal on } H \\ n-\gamma & \text{if } \chi \text{ is nonprincipal on } P \text{ and nonprincipal on } H \end{cases} \quad (3)$$

We note that Lemma 2 can be applied to almost p -ary NPS only for $\gamma = 0$ and $\gamma = -1$. Because in other cases we can not find a prime dividing $\chi(RR^{(-1)})$ for any nonprincipal character defined over a subset of G , see (3). In the cases $\gamma = 0$ or $\gamma = -1$ one can use Lemma 2 with $L = P$, and these cases are already considered in [3, Theorems 3 and 8]. We state them in Theorem 4 (i) and (ii), respectively. On the other hand, we extend Theorem 1 (iv) to almost p -ary NPS in Theorem 4 (iii).

Theorem 4. *Let p be prime number and $n \in \mathbb{Z}^+$, $\gamma \in \mathbb{Z}$ such that $|\gamma| < n$. Suppose that there exists a type γ almost p -ary NPS of period $n+1$.*

- (i) *For $\gamma = 0$, let $q \neq p$ be prime number dividing n such that $q^r \parallel n$ and q is self-conjugate modulo up for some divisor $u \geq 1$ of $n+1$. Then r is even and $q^{r/2} \leq \frac{n+1}{u}$.*
- (ii) *For $\gamma = -1$, let $q \neq p$ be prime number dividing $n-\gamma$ such that $q^r \parallel n-\gamma$ and q is self-conjugate modulo up for some divisor $u > 1$ of $n+1$. If $q \nmid u$, then r is even and $q^{r/2} \leq \frac{n+1}{u}$. If $q \mid u$ then $q^{\lfloor r/2 \rfloor} \leq 2 \frac{n+1}{u}$.*
- (iii) *For $\gamma \neq 0$, let $q \neq p$ be prime number dividing $(\gamma+1)n$ (or $n-\gamma$) such that $q^r \parallel (\gamma+1)n$ (or resp. $q^r \parallel n-\gamma$) and q is self-conjugate modulo up for some divisor $u \geq 1$ (or resp. $u > 1$) of $n+1$. If $q \nmid u$, then r is even.*

Proof. The conclusions (i) and (ii) are already proved in [3]. For the proof of (iii) we use (3). We have a nonprincipal character χ of G such that $\chi(R)\chi(R) = (\gamma + 1)n$. Hence if there exists a prime $q \neq p$ such that $q^r \mid (\gamma + 1)n$ and self-conjugate modulo up for some divisor $u \geq 1$ of $n + 1$, then by Lemma 1 r must be even. Similarly, if there exists a prime $q \neq p$ such that $q^r \mid n - \gamma$ and self-conjugate modulo up for some divisor $u > 1$ of $n + 1$, then r must be even. \square

We note that Theorem 3 (iii) can be extended to the sequences consisting of $\underline{a} = (a_0, a_1, \dots, a_{n+s-1}, \dots)$ of period $n+s$ with $a_{i_j} = 0$ for all $j = 1, 2, \dots, s$ where $\{i_1, i_2, \dots, i_j\} \subset \{0, 1, \dots, n+s-1\}$ and $a_i = \zeta_p^{b_i}$ for some integer b_i , $i \in \{0, 1, \dots, n+s-1\} \setminus \{i_1, i_2, \dots, i_j\}$ where ζ_p is a p -th root of unity in \mathbb{C} . We call \underline{a} an *almost p -ary sequence with s zero-symbols*.

Corollary 1. *Let p be prime number and $n \in \mathbb{Z}^+$, $\gamma \in \mathbb{Z}$ such that $0 < |\gamma| < n$. Let $q \neq p$ be prime number dividing $(\gamma + 1)n + (s - 1)\gamma$ (or $n - \gamma$) such that $q^r \mid (\gamma + 1)n + (s - 1)\gamma$ (or resp. $q^r \mid n - \gamma$) and q is self-conjugate modulo up for some divisor $u \geq 1$ (or resp. $u > 1$) of $n + s$ and $q \nmid u$. If there exists a type γ almost p -ary NPS of period $n + s$ with s zero-symbols, then r is even.*

Chee et al. [3] extend the results in [7] and present tables showing the existence status of almost p -ary NPS of period $n + 1$ for $\gamma = 0$ and $\gamma = 1$ and $2 \leq n \leq 100$. We extend the tables given in [3] for $|\gamma| \leq 2$ and $2 \leq n \leq 100$. In addition, we update the tables in [3] for some undecided cases. Below, we explain existence, non-existence and undecided cases. We present the detailed tables in Appendix. The empty rows in the tables are undecided cases.

For $\gamma = 0$, it is known that an $(n + 1, p, n, (n - 1)/p, 0, (n - 1)/p)$ -DPDS in $\mathbb{Z}_{n+1} \times \mathbb{Z}_p$ relative to \mathbb{Z}_{n+1} and \mathbb{Z}_p exists if n is a prime power and p divides $n - 1$ (see [10, Theorem 2.2.12]). Therefore, an almost p -ary PS of period $n + 1$ exists when n is a prime power and p is a prime divisor of $n - 1$. By using Theorem 4 and results in [3, 9] we obtain for $n \leq 100$ that an almost p -ary PS of period $n + 1$ at all other cases do not exist except the undecided pairs $(n, p) \in \{(63, 31), (77, 19), (91, 3), (92, 7), (93, 23)\}$. In Section 5, we exclude the existence at the cases $(n, p) \in \{(63, 31), (91, 3), (92, 7), (93, 23)\}$ by using multipliers.

For $\gamma = -1$, it is known that an $(n + 1, p, n, n/p, 0, n/p)$ -DPDS in $\mathbb{Z}_{n+1} \times \mathbb{Z}_p$ relative to \mathbb{Z}_{n+1} and \mathbb{Z}_p exists for $n = q - 1$ where q is a prime and p divides $q - 1$ (see [10, example 5.3.2]). Therefore, an almost p -ary NPS of period q for q is a prime and $p \mid q - 1$ exists. Theorem 2 excludes the existence of an almost p -ary NPS of period $n + 1$ for the remaining pairs $(n, p) \in \{(20, 5), (21, 7), (26, 13), (27, 3), (34, 17), (35, 5), (38, 2), (38, 19), (44, 11), (48, 3), (50, 5), (51, 3), (54, 2), (54, 3), (63, 7), (68, 17), (75, 3), (76, 19), (84, 3), (84, 7), (90, 3), (91, 7), (92, 23), (93, 31), (98, 7), (99, 11)\}$. We note that 7-ary NPS of period 77 for $\gamma = -1$ was an undecided case in [3]. Theorem 4 (ii) for $q = 3$ and $u = 2$ shows that such a sequence does not exist. It is assumed in [3] that Theorem 2 (ii) holds for $\gamma = -1$ and $u = 1$. However, we show in the proof that it does not hold for $u = 1$. Because of this reason, we say that the cases $(n, p) \in \{(38, 2), (38, 19), (50, 5), (54, 2), (54, 3), (68, 17), (84, 3), (84, 7)\}$ were undecided which are given to be nonexistent in [3].

For $\gamma = 1$, Theorem 2 excludes the existence of an almost p -ary NPS of period $n + 1$ for the pairs (n, p) such that $n \leq 100$ except the cases $(n, p) \in \{(8, 2), (8, 3), (9, 7), (14, 3), (16, 7), (18, 2), (22, 5), (23, 7), (24, 11), (25, 23), (26, 3), (32, 2), (32, 3), (32, 5), (37, 7), (38, 3), (40, 19), (44, 7), (46, 11), (48, 23), (49, 47), (50, 2), (50, 3), (54, 13), (58, 7), (62, 3), (62, 5), (64, 31), (72, 5), (72, 7), (73, 71), (74, 3), (81, 79), (82, 5), (88, 43), (90, 11), (94, 23), (95, 31), (96, 47), (98, 2), (98, 3), (100, 7)\}$. In addition, we performed an exhaustive search for the pairs $(n, p) \in \{(8, 2), (8, 3), (9, 7), (14, 3), (18, 2), (32, 2)\}$, and we obtained that an almost p -ary NPS of period $n + 1$ for $\gamma = 1$ exists for none of them.

For $\gamma = 2$, Theorem 2 excludes the existence of an almost p -ary NPS of period $n + 1$ for the pairs (n, p) such that $n \leq 100$ except the cases $(n, p) \in \{(9, 3), (12, 3), (16, 13), (24, 7), (25, 11), (26, 23), (27, 2), (27, 3), (29, 13), (33, 5), (36, 3), (36, 11), (39, 3), (47, 11), (48, 3), (48, 5), (49, 23), (50, 47), (60, 19), (63, 3), (66, 7), (69, 11), (72, 23), (74, 71), (75, 2), (75, 3), (81, 3), (81, 13), (84, 3), (93, 3), (96, 31)\}$. In addition, we exclude the existence of the pairs in $\{(9, 3), (21, 3), (27, 2)\}$ by an exhaustive search. On the other hand, we have an example of almost 3-ary NPS of period 13 for $\gamma = 2$: $(0, \zeta_3^2, \zeta_3^2, \zeta_3^2, 1, \zeta_3^2, \zeta_3, \zeta_3, \zeta_3^2, 1, \zeta_3^2, \zeta_3^2, \zeta_3^2)$.

5 Non-existence by using multipliers

An important method for the existence and the non-existence of some difference sets uses the notion of multiplier. In this section we prove the non-existence of almost p -ary PS at some values by showing the non-existence of the corresponding DPDS such that the existence of these values are not excluded by Theorem 4. We note that an $(n+1, p, n, \frac{n-\gamma-1}{p} + \gamma, 0, \frac{n-\gamma-1}{p})$ -DPDS in $G = H \times P$ relative to H and P for $\gamma = 0$ is called an $(n+1, p, n, \frac{n-1}{p})$ *relative difference set* (RDS) in $G = H \times P$ relative to P .

Let R be a subset in G . For an integer t , let $R^{(t)}$ denote the subset $R^{(t)} = \{r^t : r \in R\} \subset G$. Assume that $\gcd(t, |G|) = 1$. t is called a *multiplier* of R if there exists $g \in G$ such that

$$R^{(t)} = Rg = \{rg : r \in R\} \subset G.$$

There is a nice method for the existence and the non-existence of certain RDS that we recall here (see [3] page 406). Assume that R is an (m, n, k, λ) -RDS in G relative to P such that $k^2 \neq \lambda mn$ and t is a multiplier of R . Let Ω be the set of orbits of G under the action $x \rightarrow x^t$. Then, there exists a collection Φ of orbits (i.e. a subset $\Phi \subseteq \Omega$) such that

$$R = \bigsqcup_{A \in \Phi} A,$$

where A is an orbit in Φ . This gives a strict condition on the existence and the non-existence of RDS.

In addition, we use the following result in proving the non-existence of RDS at certain values, see [1, Lemma 5.4] or [9, Proposition 1].

Lemma 4. *Let R be an $(n+1, p, n, \frac{n-1}{p})$ -RDS in $G = \mathbb{Z}_{n+1} \times \mathbb{Z}_p$ relative to $P = \mathbb{Z}_p$. Let R have s_i many elements having i in the second component for $i = 0, 1, 2, \dots, p-1$. Then*

$$\sum_{j=0}^{p-1} s_j^2 = \frac{n(n+p-1)}{p} \text{ and } \sum_{j=0}^{p-1} s_j s_{j-i} = \frac{n(n-1)}{p}$$

for each $i = 1, 2, \dots, \lceil (p-1)/2 \rceil$, where subscripts are computed modulo p .

By using the method presented above and Lemma 4 we prove the following result.

Theorem 5. *There does not exist an almost p -ary perfect sequence with period $n+1$ for the pairs $(n, p) \in \{(63, 31), (91, 3), (92, 7), (93, 23)\}$.*

Proof. We present here the proof of the case $(91, 3)$. The others can be proven similarly. Assume that there exists an almost 3-ary PS of period 92. Using Theorem 3 we have an $(92, 3, 91, 30)$ -RDS R in $\mathbb{Z}_{92} \times \mathbb{Z}_3$ relative to \mathbb{Z}_3 . It is easy to see that $t = 13$ is a multiplier of R . Indeed let ζ be a primitive 276-th root of 1 in \mathbb{C} . We have $91 = 7 \cdot 13$ and $\zeta^{13} = (\zeta^7)^{199}$. We tabulate the set Ω of orbits under the action $x \rightarrow x^{13}$ in G in Table 1, and see that there are 12 orbits of length 1 and 24 orbits of length 11. Moreover, we may assume that there exists a subset $\Phi \subset \Omega$ satisfying

$$R = \bigsqcup_{A \in \Phi} A.$$

As in Lemma 4, let s_0, s_1 and s_2 denote the number of elements in R with the second component 0, 1 and 2 respectively. Using Lemma 4 we obtain that

$$s_0^2 + s_1^2 + s_2^2 = \frac{91 \cdot 93}{3} = 2821. \quad (4)$$

We can choose orbits of length 1 in Ω with only the same second component. Thus we can only choose at most 4 orbits of length 1 for Φ covering R . Moreover, we can not choose two orbits

A_1 and A_2 of length 11 of Ω such that with the same first components. Otherwise, difference of elements in A_1 and A_2 with the same first component gives an element in the forbidden group. We have 8 distinct subsets of orbits in Ω of length 11 with the same first component.

As $|R| = 91$, it is clear from the lengths and the numbers of the orbits in Table 1 that Φ consists of 8 distinct orbits of length 11 and 3 distinct orbits of length 1. Without loss of generality we may assume that the selected orbits of length 1 have 0 in the second component.

We conclude that $s_0 = 11k_0 + 3$, $s_1 = 11k_1$ and $s_2 = 11k_2$ for some integers k_0, k_1 and k_2 . Then, by (4) we obtain that $(11k_0 + 3)^2 + (11k_1)^2 + (11k_2)^2 = 2821$, But 2821 is not divisible by 11, which is a contradiction. □

Table 1. Orbits of $G = \mathbb{Z}_{92} \times \mathbb{Z}_3$ under $x \rightarrow x^{13}$

orbits of length 1
$\{(0, 0)\} \{(0, 1)\} \{(0, 2)\}$
$\{(23, 0)\} \{(23, 1)\} \{(23, 2)\}$
$\{(46, 0)\} \{(46, 1)\} \{(46, 2)\}$
$\{(69, 0)\} \{(69, 1)\} \{(69, 2)\}$
orbits of length 11
$\{(33, 2), (45, 2), (61, 2), (5, 2), (17, 2), (57, 2), (89, 2), (65, 2), (37, 2), (21, 2), (53, 2)\}$
$\{(31, 0), (75, 0), (35, 0), (87, 0), (55, 0), (27, 0), (47, 0), (71, 0), (3, 0), (59, 0), (39, 0)\}$
$\{(31, 2), (27, 2), (35, 2), (47, 2), (71, 2), (75, 2), (3, 2), (87, 2), (55, 2), (39, 2), (59, 2)\}$
$\{(44, 0), (56, 0), (84, 0), (88, 0), (40, 0), (76, 0), (60, 0), (20, 0), (28, 0), (80, 0), (68, 0)\}$
$\{(87, 1), (55, 1), (31, 1), (71, 1), (59, 1), (3, 1), (39, 1), (27, 1), (75, 1), (47, 1), (35, 1)\}$
$\{(50, 2), (2, 2), (26, 2), (6, 2), (62, 2), (54, 2), (58, 2), (78, 2), (70, 2), (82, 2), (18, 2)\}$
$\{(45, 0), (61, 0), (89, 0), (17, 0), (57, 0), (21, 0), (53, 0), (33, 0), (5, 0), (37, 0), (65, 0)\}$
$\{(82, 1), (70, 1), (54, 1), (62, 1), (50, 1), (6, 1), (26, 1), (18, 1), (58, 1), (2, 1), (78, 1)\}$
$\{(58, 0), (70, 0), (26, 0), (54, 0), (62, 0), (6, 0), (78, 0), (2, 0), (18, 0), (82, 0), (50, 0)\}$
$\{(36, 1), (72, 1), (32, 1), (48, 1), (52, 1), (64, 1), (24, 1), (4, 1), (12, 1), (16, 1), (8, 1)\}$
$\{(48, 2), (64, 2), (36, 2), (32, 2), (72, 2), (24, 2), (52, 2), (16, 2), (8, 2), (12, 2), (4, 2)\}$
$\{(9, 2), (73, 2), (13, 2), (81, 2), (41, 2), (1, 2), (29, 2), (49, 2), (85, 2), (25, 2), (77, 2)\}$
$\{(49, 0), (41, 0), (25, 0), (77, 0), (9, 0), (13, 0), (81, 0), (29, 0), (85, 0), (73, 0), (1, 0)\}$
$\{(76, 2), (88, 2), (60, 2), (20, 2), (40, 2), (84, 2), (28, 2), (80, 2), (56, 2), (68, 2), (44, 2)\}$
$\{(45, 1), (65, 1), (33, 1), (17, 1), (89, 1), (57, 1), (5, 1), (37, 1), (61, 1), (21, 1), (53, 1)\}$
$\{(42, 0), (86, 0), (30, 0), (66, 0), (14, 0), (10, 0), (90, 0), (22, 0), (38, 0), (74, 0), (34, 0)\}$
$\{(43, 0), (63, 0), (7, 0), (15, 0), (79, 0), (91, 0), (19, 0), (51, 0), (83, 0), (11, 0), (67, 0)\}$
$\{(63, 2), (43, 2), (91, 2), (83, 2), (19, 2), (79, 2), (67, 2), (15, 2), (51, 2), (7, 2), (11, 2)\}$
$\{(30, 2), (86, 2), (42, 2), (38, 2), (10, 2), (14, 2), (66, 2), (74, 2), (34, 2), (90, 2), (22, 2)\}$
$\{(66, 1), (90, 1), (22, 1), (30, 1), (42, 1), (14, 1), (74, 1), (34, 1), (86, 1), (10, 1), (38, 1)\}$
$\{(77, 1), (29, 1), (73, 1), (85, 1), (1, 1), (41, 1), (49, 1), (9, 1), (25, 1), (13, 1), (81, 1)\}$
$\{(36, 0), (24, 0), (72, 0), (16, 0), (8, 0), (48, 0), (64, 0), (4, 0), (32, 0), (12, 0), (52, 0)\}$
$\{(79, 1), (91, 1), (43, 1), (67, 1), (83, 1), (19, 1), (11, 1), (51, 1), (7, 1), (15, 1), (63, 1)\}$
$\{(88, 1), (76, 1), (40, 1), (44, 1), (20, 1), (28, 1), (80, 1), (68, 1), (84, 1), (56, 1), (60, 1)\}$

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A Tables

Table 2: p -ary nearly perfect sequences of period n and type $\gamma = 0$.

n	p	Comments
2	2	exists by [10, Theorem 2.2.9]
3	3	exists by [10, Theorem 2.2.9]
4	2	exists by [6, Theorem 2.3]
5	5	exists by [10, Theorem 2.2.9]
6	2	not exists by [7, Theorem 4.7] with $q=3$
7	3	not exists by [7, Corollary 4.10] with $q=3$
7	7	exists by [10, Theorem 2.2.9]
8	2	not exists by [7, Theorem 4.6] with $q=2$
9	3	exists by [6, Theorem 2.3]
10	2	not exists by [7, Theorem 4.7] with $q=5$
10	5	not exists by [7, Corollary 4.10] with $q=5$
11	11	exists by [10, Theorem 2.2.9]
12	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
13	3	not exists by Theorem 2 (i) with $q=2$ and $u=12$
13	13	exists by [10, Theorem 2.2.9]
14	2	not exists by [7, Theorem 4.7] with $q=7$
15	7	not exists by [7, Corollary 4.10] with $q=7$
15	3	not exists by [7, Theorem 4.7] with $q=5$
16	5	not exists by Theorem 2 (i) with $q=3$ and $u=1$
16	2	not exists by [7, Theorem 4.6] with $q=2$
17	17	exists by [10, Theorem 2.2.9]
18	2	not exists by Theorem 2 (i) with $q=3$ and $u=9$
19	3	not exists by [7, Corollary 4.10] with $q=3$
19	19	exists by [10, Theorem 2.2.9]
20	2	not exists by Theorem 2 (i) with $q=5$ and $u=1$
21	5	not exists by Theorem 2 (i) with $q=2$ and $u=20$
21	3	not exists by [7, Theorem 4.7] with $q=7$
22	7	not exists by Theorem 2 (i) with $q=3$ and $u=1$
22	2	not exists by [7, Theorem 4.7] with $q=11$
23	11	not exists by [7, Corollary 4.10] with $q=11$
23	23	exists by [10, Theorem 2.2.9]
24	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
25	3	not exists by Theorem 2 (i) with $q=2$ and $u=1$
25	5	exists by [6, Theorem 2.3]
26	2	not exists by [7, Theorem 4.7] with $q=13$
27	13	not exists by [7, Corollary 4.10] with $q=13$
27	3	not exists by [7, Theorem 4.6] with $q=3$
28	2	not exists by Theorem 2 (i) with $q=7$ and $u=1$
29	7	exists by [10, Theorem 2.2.9]
30	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
31	3	not exists by Theorem 2 (i) with $q=2$ and $u=1$
31	5	not exists by Theorem 2 (i) with $q=2$ and $u=1$
31	31	exists by [10, Theorem 2.2.9]
32	2	not exists by [7, Theorem 4.6] with $q=2$
33	3	not exists by [7, Theorem 4.7] with $q=11$
34	11	exists by [10, Theorem 2.2.9]
34	2	not exists by [7, Theorem 4.7] with $q=17$
35	17	not exists by [7, Corollary 4.10] with $q=17$
35	5	not exists by [7, Theorem 4.7] with $q=7$
36	7	not exists by Theorem 2 (i) with $q=5$ and $u=1$
36	2	not exists by Theorem 2 (i) with $q=3$ and $u=18$
37	3	not exists by Theorem 2 (i) with $q=2$ and $u=36$
37	37	exists by [10, Theorem 2.2.9]
38	2	not exists by [7, Theorem 4.7] with $q=19$
39	19	not exists by [7, Corollary 4.10] with $q=19$
39	3	not exists by [7, Theorem 4.7] with $q=13$
40	13	exists by [10, Theorem 2.2.9]
40	2	not exists by Theorem 2 (i) with $q=5$ and $u=1$
41	5	not exists by Theorem 2 (i) with $q=2$ and $u=1$
41	41	exists by [10, Theorem 2.2.9]
42	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
43	3	not exists by Theorem 2 (i) with $q=2$ and $u=1$
43	7	not exists by Theorem 2 (i) with $q=3$ and $u=1$
43	43	exists by [10, Theorem 2.2.9]
44	2	not exists by Theorem 2 (i) with $q=11$ and $u=1$
45	11	not exists by Theorem 2 (i) with $q=2$ and $u=44$
45	3	not exists by Theorem 2 (i) with $q=5$ and $u=1$
46	5	not exists by Theorem 2 (i) with $q=3$ and $u=45$
46	2	not exists by [7, Theorem 4.7] with $q=23$
47	23	not exists by [7, Corollary 4.10] with $q=23$
47	47	exists by [10, Theorem 2.2.9]
48	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
49	3	not exists by Theorem 2 (i) with $q=2$ and $u=16$
49	7	exists by [6, Theorem 2.3]
50	2	not exists by Theorem 2 (i) with $q=5$ and $u=25$
51	5	not exists by [7, Corollary 4.10] with $q=5$
51	3	not exists by [7, Theorem 4.7] with $q=17$
52	17	not exists by Theorem 2 (i) with $q=3$ and $u=1$
52	2	not exists by Theorem 2 (i) with $q=13$ and $u=1$
53	13	not exists by Theorem 2 (i) with $q=2$ and $u=52$
53	53	exists by [10, Theorem 2.2.9]
54	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
55	3	not exists by [7, Corollary 4.10] with $q=3$
55	5	not exists by [7, Theorem 4.7] with $q=11$

56	2	not exists by Theorem 2 (i) with $q=7$ and $u=1$
57	7	exists by [10, Theorem 2.2.9]
57	3	not exists by [7, Theorem 4.7] with $q=19$
58	19	not exists by Theorem 2 (i) with $q=3$ and $u=1$
58	2	not exists by [7, Theorem 4.7] with $q=29$
59	29	not exists by [7, Corollary 4.10] with $q=29$
59	59	exists by [10, Theorem 2.2.9]
60	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
61	3	not exists by Theorem 2 (i) with $q=5$ and $u=1$
61	5	not exists by Theorem 2 (i) with $q=3$ and $u=1$
61	61	exists by [10, Theorem 2.2.9]
62	2	not exists by [7, Theorem 4.7] with $q=31$
63	31	not exists by [7, Corollary 4.10] with $q=31$
63	3	exists by [10, Theorem 2.2.9]
64	7	not exists by Theorem 2 (i) with $q=3$ and $u=63$
64	2	not exists by [7, Theorem 4.6] with $q=2$
65	5	not exists by [7, Theorem 4.7] with $q=13$
66	13	not exists by Theorem 2 (i) with $q=5$ and $u=1$
66	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
67	3	not exists by Theorem 2 (i) with $q=2$ and $u=1$
67	11	not exists by Theorem 2 (i) with $q=2$ and $u=1$
67	67	exists by [10, Theorem 2.2.9]
68	2	not exists by Theorem 2 (i) with $q=17$ and $u=1$
69	17	not exists by Theorem 2 (i) with $q=2$ and $u=68$
69	3	not exists by [7, Theorem 4.7] with $q=23$
70	23	exists by [10, Theorem 2.2.9]
70	2	not exists by Theorem 2 (i) with $q=5$ and $u=1$
71	5	not exists by Theorem 2 (i) with $q=2$ and $u=1$
71	7	not exists by Theorem 2 (i) with $q=5$ and $u=1$
71	71	exists by [10, Theorem 2.2.9]
72	2	not exists by [5, Result 2.3]
73	3	not exists by Theorem 2 (i) with $q=2$ and $u=1$
73	73	exists by [10, Theorem 2.2.9]
74	2	not exists by [7, Theorem 4.7] with $q=37$
75	37	not exists by [7, Corollary 4.10] with $q=37$
75	3	not exists by Theorem 2 (i) with $q=5$ and $u=25$
76	5	not exists by Theorem 2 (i) with $q=3$ and $u=1$
76	2	not exists by Theorem 2 (i) with $q=19$ and $u=1$
77	19	not exists by Theorem 2 (i) with $q=2$ and $u=76$
77	7	not exists by [7, Theorem 4.7] with $q=11$
78	11	not exists by Theorem 2 (i) with $q=7$ and $u=1$
78	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
79	3	not exists by Theorem 2 (i) with $q=2$ and $u=1$
79	13	not exists by Theorem 2 (i) with $q=2$ and $u=1$
79	79	exists by [10, Theorem 2.2.9]
80	2	not exists by Theorem 2 (i) with $q=5$ and $u=1$
81	5	not exists by Theorem 2 (i) with $q=2$ and $u=40$
81	3	not exists by [7, Theorem 4.6] with $q=3$
82	2	not exists by [7, Theorem 4.7] with $q=41$
83	41	not exists by [7, Corollary 4.10] with $q=41$
83	83	exists by [10, Theorem 2.2.9]
84	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
85	3	not exists by Theorem 2 (i) with $q=3$ and $u=1$
85	7	not exists by [7, Theorem 4.7] with $q=17$
86	17	not exists by Theorem 2 (i) with $q=5$ and $u=1$
86	2	not exists by [7, Theorem 4.7] with $q=43$
87	43	not exists by [7, Corollary 4.10] with $q=43$
87	3	not exists by [7, Theorem 4.7] with $q=29$
88	29	not exists by Theorem 2 (i) with $q=3$ and $u=1$
88	2	not exists by Theorem 2 (i) with $q=11$ and $u=1$
89	11	not exists by Theorem 2 (i) with $q=2$ and $u=1$
89	89	exists by [10, Theorem 2.2.9]
90	2	not exists by Theorem 2 (i) with $q=5$ and $u=1$
91	3	not exists by Theorem 2 (i) with $q=2$ and $u=1$
91	5	not exists by Theorem 2 (i) with $q=2$ and $u=1$
91	7	not exists by [7, Theorem 4.7] with $q=13$
92	13	not exists by Theorem 2 (i) with $q=7$ and $u=1$
92	2	not exists by Theorem 2 (i) with $q=23$ and $u=1$
93	23	exists by [10, Theorem 2.2.9]
93	3	not exists by [7, Theorem 4.7] with $q=31$
94	31	not exists by Theorem 2 (i) with $q=3$ and $u=1$
94	2	not exists by [7, Theorem 4.7] with $q=47$
95	47	not exists by [7, Corollary 4.10] with $q=47$
95	5	not exists by [7, Theorem 4.7] with $q=19$
96	19	exists by [10, Theorem 2.2.9]
96	2	not exists by Theorem 2 (i) with $q=3$ and $u=1$
97	3	not exists by Theorem 2 (i) with $q=2$ and $u=1$
97	97	exists by [10, Theorem 2.2.9]
98	2	not exists by Theorem 2 (i) with $q=7$ and $u=49$
99	7	not exists by [7, Corollary 4.10] with $q=7$
99	3	not exists by Theorem 2 (i) with $q=11$ and $u=1$
100	11	exists by [10, Theorem 2.2.9]
100	2	not exists by Theorem 2 (i) with $q=5$ and $u=25$
101	5	not exists by Theorem 2 (i) with $q=2$ and $u=100$

Table 3: p -ary nearly perfect sequences of period n and type $\gamma = -1$.

n	p	Comments
2	3	exists by [4]
3	2	exists by [5, Corollary 2.8]
4	5	exists by [4]
5	2	not exists by Theorem 2 (ii) with $q=3$ and $u=5$
6	3	exists and given in [7]
7	2	exists by [4]
8	3	exists by [5, Corollary 2.8]
9	2	not exists by Theorem 2 (ii) with $q=5$ and $u=3$
10	5	not exists by Theorem 2 (iii) with $q=5$ and $u=9$
11	11	exists by [4]
12	2	exists by [5, Corollary 2.8]
13	3	not exists by Theorem 2 (ii) with $q=2$ and $u=11$
14	13	exists by [4]
15	2	not exists by Theorem 2 (ii) with $q=7$ and $u=13$
16	3	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
17	5	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
18	2	exists by [5, Corollary 2.8]
19	17	exists by [4]
20	2	not exists by Theorem 2 (ii) with $q=3$ and $u=17$
21	3	not exists by Theorem 2 (iii) with $q=3$ and $u=17$
22	19	exists by [4]
23	2	exists by [5, Corollary 2.8]
24	3	
25	5	exists by [4]
26	2	not exists by Theorem 2 (ii) with $q=13$ and $u=5$
27	13	not exists by Theorem 2 (ii) with $q=2$ and $u=5$
28	3	exists by [4]
29	7	not exists by an exhaustive search
30	29	exists by [4]
31	2	not exists by Theorem 2 (ii) with $q=3$ and $u=29$
32	3	not exists by Theorem 2 (ii) with $q=5$ and $u=29$
33	5	not exists by Theorem 2 (ii) with $q=2$ and $u=29$
34	31	exists by [4]
35	2	exists by [5, Corollary 2.8]
36	3	not exists by Theorem 2 (ii) with $q=11$ and $u=2$
37	11	
38	2	not exists by Theorem 2 (ii) with $q=17$ and $u=3$
39	17	not exists by Theorem 2 (iii) with $q=17$ and $u=11$
40	5	not exists by Theorem 2 (ii) with $q=7$ and $u=2$
41	7	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
42	2	exists by [5, Corollary 2.8]
43	3	
44	37	exists by [4]
45	2	not exists by Theorem 2 (ii) with $q=19$ and $u=37$
46	19	not exists by Theorem 2 (iii) with $q=19$ and $u=37$
47	38	
48	13	not exists by Theorem 2 (iii) with $q=13$ and $u=19$
49	2	not exists by Theorem 2 (ii) with $q=5$ and $u=3$
50	5	not exists by Theorem 2 (ii) with $q=2$ and $u=13$
51	41	exists by [4]
52	2	not exists by Theorem 2 (ii) with $q=3$ and $u=41$
53	3	
54	7	not exists by Theorem 2 (iii) with $q=7$ and $u=41$
55	43	exists by [4]
56	11	exists by [5, Corollary 2.8]
57	11	not exists by Theorem 2 (ii) with $q=2$ and $u=43$
58	43	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
59	2	not exists by Theorem 2 (ii) with $q=23$ and $u=3$
60	23	not exists by Theorem 2 (iii) with $q=23$ and $u=9$
61	47	exists by [4]
62	2	exists by [5, Corollary 2.8]
63	2	exists by [5, Corollary 2.8]
64	2	exists by [5, Corollary 2.8]
65	2	exists by [5, Corollary 2.8]
66	2	exists by [5, Corollary 2.8]
67	2	exists by [5, Corollary 2.8]
68	2	exists by [5, Corollary 2.8]
69	2	exists by [5, Corollary 2.8]
70	2	exists by [5, Corollary 2.8]
71	2	exists by [5, Corollary 2.8]
72	2	exists by [5, Corollary 2.8]
73	2	exists by [5, Corollary 2.8]
74	2	exists by [5, Corollary 2.8]
75	2	exists by [5, Corollary 2.8]
76	2	exists by [5, Corollary 2.8]
77	2	exists by [5, Corollary 2.8]
78	2	exists by [5, Corollary 2.8]
79	2	exists by [5, Corollary 2.8]
80	2	exists by [5, Corollary 2.8]
81	2	exists by [5, Corollary 2.8]
82	2	exists by [5, Corollary 2.8]
83	2	exists by [5, Corollary 2.8]
84	2	exists by [5, Corollary 2.8]
85	2	exists by [5, Corollary 2.8]
86	2	exists by [5, Corollary 2.8]
87	2	exists by [5, Corollary 2.8]
88	2	exists by [5, Corollary 2.8]
89	2	exists by [5, Corollary 2.8]
90	2	exists by [5, Corollary 2.8]
91	2	exists by [5, Corollary 2.8]
92	2	exists by [5, Corollary 2.8]
93	2	exists by [5, Corollary 2.8]
94	2	exists by [5, Corollary 2.8]
95	2	exists by [5, Corollary 2.8]
96	2	exists by [5, Corollary 2.8]
97	2	exists by [5, Corollary 2.8]
98	2	exists by [5, Corollary 2.8]
99	2	exists by [5, Corollary 2.8]
100	101	exists by [4]

55	2	not exists by Theorem 2 (ii) with $q=7$ and $u=5$
56	3	
57	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
58	29	not exists by Theorem 2 (iii) with $q=29$ and $u=3$
59	2	exists by [5, Corollary 2.8]
60	3	not exists by Theorem 2 (ii) with $q=2$ and $u=59$
61	61	exists by [4]
62	2	not exists by Theorem 2 (ii) with $q=31$ and $u=61$
63	3	not exists by Theorem 2 (iii) with $q=31$ and $u=61$
64	3	not exists by Theorem 2 (iii) with $q=3$ and $u=62$
65	7	not exists by Theorem 2 (ii) with $q=3$ and $u=31$
66	2	exists by [4]
67	5	not exists by Theorem 2 (ii) with $q=13$ and $u=2$
68	13	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
69	2	not exists by Theorem 2 (ii) with $q=3$ and $u=5$
70	3	
71	11	
72	67	exists by [4]
73	2	exists by [5, Corollary 2.8]
74	17	
75	3	not exists by Theorem 2 (ii) with $q=23$ and $u=2$
76	23	not exists by Theorem 2 (iii) with $q=23$ and $u=17$
77	2	not exists by Theorem 2 (ii) with $q=5$ and $u=3$
78	5	not exists by Theorem 2 (iii) with $q=5$ and $u=69$
79	7	not exists by Theorem 2 (ii) with $q=5$ and $u=3$
80	71	exists by [4]
81	2	exists by [5, Corollary 2.8]
82	3	
83	73	exists by [4]
84	2	not exists by [5, Corollary 2.8]
85	37	
86	3	
87	5	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
88	2	not exists by Theorem 2 (ii) with $q=19$ and $u=5$
89	19	not exists by Theorem 2 (iii) with $q=19$ and $u=25$
90	76	
91	11	not exists by Theorem 2 (ii) with $q=7$ and $u=2$
92	2	not exists by Theorem 2 (ii) with $q=3$ and $u=7$
93	3	not exists by Theorem 2 (ii) with $q=2$ and $u=11$
94	13	not exists by Theorem 2 (iii) with $q=13$ and $u=77$
95	78	exists by [4]
96	79	exists by [5, Corollary 2.8]
97	2	exists by [5, Corollary 2.8]
98	5	
99	3	exists by [4]
100	2	not exists by Theorem 2 (ii) with $q=41$ and $u=3$
101	41	not exists by Theorem 2 (iii) with $q=41$ and $u=9$
102	83	exists by [4]
103	2	exists by [5, Corollary 2.8]
104	3	not exists by Theorem 2 (ii) with $q=2$ and $u=83$
105	7	
106	5	not exists by Theorem 2 (ii) with $q=17$ and $u=2$
107	17	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
108	2	not exists by Theorem 2 (ii) with $q=43$ and $u=5$
109	43	not exists by Theorem 2 (iii) with $q=43$ and $u=17$
110	86	not exists by Theorem 2 (ii) with $q=29$ and $u=2$
111	29	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
112	87	not exists by Theorem 2 (ii) with $q=11$ and $u=3$
113	11	not exists by Theorem 2 (ii) with $q=2$ and $u=3$
114	88	exists by [4]
115	89	not exists by Theorem 2 (ii) with $q=3$ and $u=89$
116	3	not exists by Theorem 2 (iii) with $q=3$ and $u=89$
117	5	not exists by Theorem 2 (iii) with $q=5$ and $u=89$
118	90	not exists by Theorem 2 (ii) with $q=13$ and $u=2$
119	13	not exists by Theorem 2 (ii) with $q=7$ and $u=2$
120	91	not exists by Theorem 2 (ii) with $q=23$ and $u=13$
121	23	
122	92	
123	31	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
124	2	not exists by Theorem 2 (ii) with $q=47$ and $u=3$
125	47	
126	5	not exists by Theorem 2 (ii) with $q=19$ and $u=2$
127	19	not exists by Theorem 2 (iii) with $q=19$ and $u=47$
128	95	not exists by Theorem 2 (ii) with $q=3$ and $u=5$
129	3	not exists by Theorem 2 (ii) with $q=2$ and $u=19$
130	96	exists by [4]
131	97	not exists by Theorem 2 (ii) with $q=7$ and $u=97$
132	7	not exists by Theorem 2 (iii) with $q=7$ and $u=97$
133	98	not exists by Theorem 2 (ii) with $q=11$ and $u=2$
134	11	
135	99	exists by [5, Corollary 2.8]
136	5	
137	100	exists by [4]

Table 4: p -ary nearly perfect sequences of period n and type $\gamma = 1$.

n	p	Comments
3	2	not exists by Theorem 2 (iv) with $q=5$ and $u=1$
4	3	
5	2	exists by an exhaustive search
6	5	not exists by Theorem 2 (iii) with $q=5$ and $u=6$
7	2	not exists by Theorem 2 (ii) with $q=3$ and $u=7$
8	3	
9	7	not exists by Theorem 2 (iii) with $q=7$ and $u=8$
10	2	not exists by Theorem 2 (iv) with $q=17$ and $u=1$
11	3	not exists by Theorem 2 (iii) with $q=3$ and $u=10$
12	2	not exists by Theorem 2 (iv) with $q=3$ and $u=1$
13	5	not exists by Theorem 2 (iv) with $q=3$ and $u=1$
14	11	not exists by Theorem 2 (iii) with $q=11$ and $u=6$
15	2	exists by an exhaustive search
16	3	exists and given in [7]
17	13	not exists by Theorem 2 (iii) with $q=13$ and $u=7$
18	2	not exists by Theorem 2 (ii) with $q=7$ and $u=5$
19	7	
20	3	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
21	5	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
22	2	not exists by Theorem 2 (iii) with $q=2$ and $u=17$
23	17	not exists by Theorem 2 (iv) with $q=17$ and $u=6$
24	2	not exists by Theorem 2 (ii) with $q=3$ and $u=19$
25	3	not exists by Theorem 2 (ii) with $q=2$ and $u=19$
26	19	not exists by Theorem 2 (iii) with $q=19$ and $u=5$
27	2	not exists by Theorem 2 (ii) with $q=5$ and $u=3$
28	5	not exists by Theorem 2 (iii) with $q=5$ and $u=21$
29	3	
30	7	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
31	2	not exists by Theorem 2 (ii) with $q=11$ and $u=23$
32	11	not exists by Theorem 2 (iii) with $q=11$ and $u=23$
33	23	not exists by Theorem 2 (iii) with $q=23$ and $u=6$
34	2	not exists by Theorem 2 (ii) with $q=3$ and $u=5$
35	3	
36	5	not exists by Theorem 2 (iii) with $q=5$ and $u=13$
37	2	not exists by Theorem 2 (iv) with $q=53$ and $u=1$
38	13	
39	3	not exists by Theorem 2 (iii) with $q=3$ and $u=14$
40	2	not exists by Theorem 2 (iv) with $q=3$ and $u=1$
41	7	not exists by Theorem 2 (iv) with $q=3$ and $u=1$
42	29	not exists by Theorem 2 (iii) with $q=29$ and $u=5$
43	31	not exists by Theorem 2 (ii) with $q=3$ and $u=31$
44	3	
45	5	
46	31	not exists by Theorem 2 (iii) with $q=31$ and $u=8$
47	2	not exists by Theorem 2 (iii) with $q=2$ and $u=33$
48	3	not exists by Theorem 2 (ii) with $q=11$ and $u=2$
49	11	not exists by Theorem 2 (iii) with $q=11$ and $u=17$
50	2	not exists by Theorem 2 (ii) with $q=17$ and $u=5$
51	17	not exists by Theorem 2 (iv) with $q=3$ and $u=1$
52	3	not exists by Theorem 2 (iii) with $q=3$ and $u=41$
53	2	not exists by Theorem 2 (ii) with $q=7$ and $u=2$
54	7	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
55	37	not exists by Theorem 2 (ii) with $q=3$ and $u=37$
56	3	not exists by Theorem 2 (iii) with $q=3$ and $u=37$
57	37	not exists by Theorem 2 (iii) with $q=37$ and $u=19$
58	2	not exists by Theorem 2 (ii) with $q=19$ and $u=13$
59	19	not exists by Theorem 2 (ii) with $q=2$ and $u=3$
60	40	
61	13	
62	41	not exists by Theorem 2 (ii) with $q=5$ and $u=41$
63	5	not exists by Theorem 2 (ii) with $q=2$ and $u=41$
64	41	not exists by Theorem 2 (iii) with $q=41$ and $u=6$
65	43	not exists by Theorem 2 (ii) with $q=3$ and $u=43$
66	3	not exists by Theorem 2 (ii) with $q=2$ and $u=43$
67	7	not exists by Theorem 2 (ii) with $q=3$ and $u=43$
68	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
69	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
70	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
71	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
72	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
73	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
74	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
75	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
76	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
77	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
78	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
79	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
80	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
81	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
82	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
83	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
84	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
85	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
86	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
87	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
88	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
89	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
90	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
91	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
92	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
93	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
94	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
95	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
96	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
97	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
98	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
99	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$
100	43	not exists by Theorem 2 (iii) with $q=43$ and $u=11$

Table 5: p -ary nearly perfect sequences of period n and type $\gamma = 2$.

n	p	Comments
4	2	not exists by Theorem 2 (iv) with $q=5$ and $u=1$
5	3	exists and given in Section 3
6	2	exists an exhaustive search
7	5	not exists by Theorem 2 (iii) with $q=5$ and $u=7$
8	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
9	3	not exists by Theorem 2 (iv) with $q=2$ and $u=1$
9	7	not exists by an exhaustive search
10	2	not exists by Theorem 2 (iv) with $q=7$ and $u=1$
11	3	
12	2	not exists by Theorem 2 (ii) with $q=5$ and $u=3$
13	5	not exists by Theorem 2 (iv) with $q=2$ and $u=1$
13	11	not exists by Theorem 2 (iii) with $q=11$ and $u=13$
14	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
15	3	not exists by Theorem 2 (iv) with $q=2$ and $u=1$
15	13	not exists by Theorem 2 (iii) with $q=13$ and $u=5$
16	2	not exists by Theorem 2 (ii) with $q=7$ and $u=2$
17	3	exists and given in Section 3
18	5	not exists by Theorem 2 (iii) with $q=5$ and $u=17$
18	2	not exists by Theorem 2 (iv) with $q=13$ and $u=1$
19	17	not exists by Theorem 2 (iv) with $q=5$ and $u=1$
20	2	not exists by Theorem 2 (iv) with $q=29$ and $u=1$
21	3	not exists by Theorem 2 (iv) with $q=2$ and $u=1$
21	19	not exists by Theorem 2 (iii) with $q=19$ and $u=7$
22	2	not exists by [5, Corollary 2.6]
23	5	
23	3	
24	7	not exists by Theorem 2 (iii) with $q=7$ and $u=23$
24	2	not exists by Theorem 2 (ii) with $q=11$ and $u=2$
25	11	not exists by Theorem 2 (ii) with $q=2$ and $u=3$
25	23	not exists by Theorem 2 (iii) with $q=23$ and $u=5$
26	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
27	3	
27	5	not exists by Theorem 2 (iii) with $q=5$ and $u=27$
28	2	not exists by Theorem 2 (ii) with $q=13$ and $u=7$
29	13	not exists by Theorem 2 (iii) with $q=13$ and $u=14$
29	3	not exists by Theorem 2 (iii) with $q=3$ and $u=29$
30	2	not exists by Theorem 2 (ii) with $q=7$ and $u=2$
31	7	
31	29	not exists by Theorem 2 (iii) with $q=29$ and $u=31$
32	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
33	3	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
33	5	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
33	31	
34	2	not exists by [5, Corollary 2.6]
35	3	
36	11	
36	2	not exists by Theorem 2 (ii) with $q=17$ and $u=3$
37	17	not exists by Theorem 2 (iii) with $q=17$ and $u=9$
37	5	not exists by Theorem 2 (iii) with $q=5$ and $u=37$
38	7	
38	2	not exists by Theorem 2 (ii) with $q=3$ and $u=19$
39	3	not exists by Theorem 2 (iii) with $q=3$ and $u=38$
39	37	not exists by Theorem 2 (iii) with $q=37$ and $u=13$
40	2	not exists by Theorem 2 (ii) with $q=19$ and $u=2$
41	19	not exists by Theorem 2 (iii) with $q=19$ and $u=10$
41	3	
42	13	not exists by Theorem 2 (iii) with $q=13$ and $u=41$
42	2	not exists by Theorem 2 (ii) with $q=5$ and $u=3$
43	5	not exists by Theorem 2 (iii) with $q=5$ and $u=42$
43	41	not exists by Theorem 2 (iv) with $q=127$ and $u=1$
44	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
44	3	not exists by Theorem 2 (ii) with $q=2$ and $u=11$
45	7	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
45	43	not exists by Theorem 2 (iii) with $q=43$ and $u=5$
46	2	not exists by Theorem 2 (ii) with $q=11$ and $u=2$
47	11	not exists by Theorem 2 (iii) with $q=11$ and $u=23$
47	3	not exists by Theorem 2 (ii) with $q=5$ and $u=47$
48	5	not exists by Theorem 2 (iii) with $q=5$ and $u=47$
48	2	not exists by Theorem 2 (ii) with $q=23$ and $u=2$
49	23	not exists by Theorem 2 (iii) with $q=23$ and $u=12$
49	47	not exists by Theorem 2 (iii) with $q=47$ and $u=7$
50	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
51	3	
51	7	not exists by Theorem 2 (iii) with $q=7$ and $u=17$
52	2	not exists by Theorem 2 (ii) with $q=5$ and $u=13$
53	5	not exists by Theorem 2 (ii) with $q=2$ and $u=13$
53	3	not exists by Theorem 2 (ii) with $q=17$ and $u=53$
54	17	not exists by Theorem 2 (iii) with $q=17$ and $u=53$
54	2	not exists by Theorem 2 (iv) with $q=5$ and $u=1$
55	13	not exists by Theorem 2 (iv) with $q=2$ and $u=1$
55	53	not exists by Theorem 2 (ii) with $q=53$ and $u=5$
56	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$

57	3	not exists by Theorem 2 (iii) with $q=3$ and $u=28$
57	5	
58	11	
58	2	not exists by Theorem 2 (ii) with $q=7$ and $u=2$
59	7	
59	3	
59	19	
60	2	not exists by Theorem 2 (ii) with $q=29$ and $u=3$
61	29	not exists by Theorem 2 (ii) with $q=2$ and $u=5$
61	59	not exists by Theorem 2 (iii) with $q=59$ and $u=61$
62	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
62	3	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
63	5	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
63	61	not exists by Theorem 2 (iii) with $q=61$ and $u=7$
64	2	not exists by Theorem 2 (ii) with $q=31$ and $u=2$
65	31	not exists by Theorem 2 (iii) with $q=31$ and $u=16$
66	3	
66	7	not exists by Theorem 2 (iii) with $q=7$ and $u=65$
66	2	not exists by Theorem 2 (iii) with $q=2$ and $u=33$
67	5	not exists by Theorem 2 (iii) with $q=5$ and $u=67$
67	13	not exists by Theorem 2 (iii) with $q=13$ and $u=67$
68	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
68	3	not exists by Theorem 2 (ii) with $q=11$ and $u=2$
69	11	not exists by Theorem 2 (iii) with $q=11$ and $u=34$
69	67	not exists by Theorem 2 (iii) with $q=67$ and $u=23$
70	2	not exists by Theorem 2 (ii) with $q=17$ and $u=5$
71	17	not exists by Theorem 2 (iv) with $q=13$ and $u=1$
71	3	not exists by Theorem 2 (ii) with $q=23$ and $u=71$
71	23	not exists by Theorem 2 (iii) with $q=23$ and $u=71$
72	2	not exists by Theorem 2 (ii) with $q=5$ and $u=3$
72	5	not exists by Theorem 2 (ii) with $q=7$ and $u=2$
73	7	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
73	71	not exists by Theorem 2 (iii) with $q=71$ and $u=73$
74	2	not exists by Theorem 2 (ii) with $q=3$ and $u=37$
75	3	not exists by Theorem 2 (iii) with $q=3$ and $u=74$
75	73	not exists by Theorem 2 (iii) with $q=73$ and $u=5$
76	2	not exists by Theorem 2 (ii) with $q=37$ and $u=19$
77	37	not exists by Theorem 2 (iii) with $q=37$ and $u=19$
77	3	
78	5	not exists by Theorem 2 (iv) with $q=229$ and $u=1$
78	2	not exists by Theorem 2 (ii) with $q=19$ and $u=2$
79	19	not exists by Theorem 2 (iii) with $q=19$ and $u=26$
79	7	not exists by Theorem 2 (iii) with $q=7$ and $u=79$
80	11	not exists by Theorem 2 (ii) with $q=7$ and $u=79$
80	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
81	3	not exists by Theorem 2 (iv) with $q=2$ and $u=1$
81	13	not exists by Theorem 2 (ii) with $q=2$ and $u=5$
81	79	
82	2	not exists by Theorem 2 (ii) with $q=5$ and $u=41$
82	5	not exists by Theorem 2 (ii) with $q=2$ and $u=41$
83	3	
84	2	not exists by Theorem 2 (ii) with $q=41$ and $u=3$
85	41	not exists by Theorem 2 (iii) with $q=41$ and $u=14$
85	83	not exists by Theorem 2 (iii) with $q=83$ and $u=5$
86	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
87	3	
87	7	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
87	5	not exists by Theorem 2 (ii) with $q=17$ and $u=29$
88	17	not exists by Theorem 2 (iii) with $q=17$ and $u=29$
88	2	not exists by Theorem 2 (ii) with $q=43$ and $u=2$
89	43	not exists by Theorem 2 (ii) with $q=2$ and $u=11$
89	3	not exists by Theorem 2 (iv) with $q=5$ and $u=1$
90	29	not exists by Theorem 2 (iii) with $q=29$ and $u=89$
90	2	not exists by Theorem 2 (ii) with $q=11$ and $u=2$
91	11	not exists by Theorem 2 (ii) with $q=2$ and $u=3$
91	89	not exists by Theorem 2 (iii) with $q=89$ and $u=7$
92	2	not exists by Theorem 2 (ii) with $q=5$ and $u=23$
92	3	not exists by Theorem 2 (ii) with $q=5$ and $u=2$
93	5	not exists by Theorem 2 (iv) with $q=2$ and $u=1$
93	7	not exists by Theorem 2 (ii) with $q=13$ and $u=31$
94	13	not exists by Theorem 2 (iii) with $q=13$ and $u=31$
94	2	not exists by Theorem 2 (ii) with $q=23$ and $u=2$
95	23	not exists by Theorem 2 (iii) with $q=23$ and $u=47$
95	3	
96	31	not exists by Theorem 2 (ii) with $q=3$ and $u=19$
96	2	not exists by Theorem 2 (ii) with $q=47$ and $u=2$
97	47	not exists by Theorem 2 (iii) with $q=47$ and $u=12$
97	5	not exists by Theorem 2 (iii) with $q=5$ and $u=97$
98	19	not exists by Theorem 2 (iii) with $q=19$ and $u=97$
98	2	not exists by Theorem 2 (ii) with $q=3$ and $u=2$
99	3	
99	97	not exists by Theorem 2 (iv) with $q=5$ and $u=1$
100	2	not exists by Theorem 2 (ii) with $q=7$ and $u=25$
100	7	not exists by Theorem 2 (iii) with $q=7$ and $u=50$

Table 6: almost p -ary nearly perfect sequences of period $n + 1$ and type $\gamma = 0$.

n	p	Comments
3	2	not exists by [3, Theorem 2]
4	3	exists by [3, Example 1]
5	2	not exists by [3, Theorem 2]
6	5	not exists by [3, Result 5]
7	2	not exists by [3, Theorem 2]
8	3	exists by [3, Example 1]
9	7	exists by [3, Example 1]
9	2	not exists by [3, Theorem 2]
10	3	not exists by Theorem 4 (i) with $q=2$ and $u=1$
11	2	not exists by [3, Theorem 2]
11	5	exists by [3, Example 1]
12	11	not exists by [3, Result 5]
13	2	not exists by [3, Theorem 2]
13	3	exists by [3, Example 1]
14	13	not exists by [3, Result 5]
15	2	not exists by [3, Theorem 2]
15	7	not exists by Theorem 4 (i) with $q=3$ and $u=1$
16	3	exists by [3, Example 1]
16	5	exists by [3, Example 1]
17	2	not exists by [3, Theorem 2]
18	17	not exists by [3, Result 5]
19	2	not exists by [3, Theorem 2]
19	3	exists by [3, Example 1]
20	19	not exists by [3, Result 5]
21	2	not exists by [3, Theorem 2]
21	5	not exists by Theorem 4 (i) with $q=3$ and $u=1$
22	3	not exists by Theorem 4 (i) with $q=2$ and $u=1$
22	7	not exists by [3, Theorem 5]
23	2	not exists by [3, Theorem 2]
23	11	exists by [3, Example 1]
24	23	not exists by [3, Result 5]
25	2	not exists by [3, Theorem 2]
25	3	exists by [3, Example 1]
26	5	not exists by Theorem 4 (i) with $q=2$ and $u=1$
27	2	not exists by [3, Theorem 2]
27	13	exists by [3, Example 1]
28	3	not exists by [3, Theorem 5]
29	2	not exists by [3, Theorem 2]
29	7	exists by [3, Example 1]
30	29	not exists by [3, Result 5]
31	2	not exists by [3, Theorem 2]
31	3	exists by [3, Example 1]
31	5	exists by [3, Example 1]
32	31	exists by [3, Example 1]
33	2	not exists by [3, Theorem 2]
34	3	not exists by Theorem 4 (i) with $q=2$ and $u=1$
34	11	not exists by Theorem 4 (i) with $q=2$ and $u=1$
35	2	not exists by [3, Theorem 2]
35	17	not exists by Theorem 4 (i) with $q=5$ and $u=1$
36	5	not exists by Theorem 4 (i) with $q=2$ and $u=37$
36	7	not exists by Theorem 4 (i) with $q=3$ and $u=37$
37	2	not exists by [3, Theorem 2]
37	3	exists by [3, Example 1]
38	37	not exists by [3, Result 5]
39	2	not exists by [3, Theorem 2]
39	19	not exists by Theorem 4 (i) with $q=3$ and $u=1$
40	3	not exists by Theorem 4 (i) with $q=2$ and $u=1$
40	13	not exists by Theorem 4 (i) with $q=2$ and $u=1$
41	2	not exists by [3, Theorem 2]
41	5	exists by [3, Example 1]
42	41	not exists by [3, Result 5]
43	2	not exists by [3, Theorem 2]
43	3	exists by [3, Example 1]
43	7	exists by [3, Example 1]
44	43	not exists by [3, Result 5]
45	2	not exists by [3, Theorem 2]
45	11	not exists by [3, Theorem 5]
46	3	not exists by Theorem 4 (i) with $q=2$ and $u=1$
46	5	not exists by Theorem 4 (i) with $q=2$ and $u=1$
47	2	not exists by [3, Theorem 2]
47	23	exists by [3, Example 1]
48	47	not exists by [3, Result 5]
49	2	not exists by [3, Theorem 2]
49	3	exists by [3, Example 1]
50	7	not exists by [9]
51	2	not exists by [3, Theorem 2]
51	5	not exists by Theorem 4 (i) with $q=3$ and $u=1$
52	3	not exists by [3, Theorem 5]
52	17	not exists by Theorem 4 (i) with $q=13$ and $u=1$
53	2	not exists by [3, Theorem 2]
53	13	exists by [3, Example 1]
54	53	not exists by [3, Result 5]
55	2	not exists by [3, Theorem 2]
55	3	not exists by Theorem 4 (i) with $q=5$ and $u=1$
56	5	not exists by Theorem 4 (i) with $q=2$ and $u=1$
56	11	not exists by Theorem 4 (i) with $q=2$ and $u=1$
57	2	not exists by [3, Theorem 2]
57	7	not exists by Theorem 4 (i) with $q=3$ and $u=1$
58	3	not exists by Theorem 4 (i) with $q=2$ and $u=1$
58	19	not exists by Theorem 4 (i) with $q=2$ and $u=1$
59	2	not exists by [3, Theorem 2]
59	29	exists by [3, Example 1]
60	59	not exists by [3, Result 5]
61	2	not exists by [3, Theorem 2]
61	3	exists by [3, Example 1]
61	5	exists by [3, Example 1]
62	61	not exists by [3, Result 5]
63	2	not exists by [3, Theorem 2]
63	31	not exists by Theorem 5
64	3	exists by [3, Example 1]
64	7	exists by [3, Example 1]
65	2	not exists by [3, Theorem 2]
66	5	not exists by Theorem 4 (i) with $q=2$ and $u=1$
66	13	not exists by Theorem 4 (i) with $q=2$ and $u=1$
67	2	not exists by [3, Theorem 2]
67	3	exists by [3, Example 1]
67	11	exists by [3, Example 1]
68	67	not exists by [3, Result 5]
69	2	not exists by [3, Theorem 2]
69	17	not exists by Theorem 4 (i) with $q=3$ and $u=1$
70	3	not exists by Theorem 4 (i) with $q=2$ and $u=1$
70	23	not exists by Theorem 4 (i) with $q=5$ and $u=1$
71	2	not exists by [3, Theorem 2]
71	5	exists by [3, Example 1]
71	7	exists by [3, Example 1]
72	71	not exists by [3, Result 5]
73	2	not exists by [3, Theorem 2]
73	3	exists by [3, Example 1]
74	73	not exists by [3, Result 5]
75	2	not exists by [3, Theorem 2]
75	37	not exists by Theorem 4 (i) with $q=3$ and $u=1$
76	3	not exists by [9]
76	5	not exists by Theorem 4 (i) with $q=19$ and $u=1$
77	2	not exists by [3, Theorem 2]
77	19	
78	7	not exists by Theorem 4 (i) with $q=3$ and $u=1$
78	11	not exists by Theorem 4 (i) with $q=2$ and $u=1$
79	2	not exists by [3, Theorem 2]
79	3	exists by [3, Example 1]
79	13	exists by [3, Example 1]
80	79	not exists by [3, Result 5]
81	2	not exists by [3, Theorem 2]
81	5	exists by [3, Example 1]
82	3	not exists by Theorem 4 (i) with $q=2$ and $u=1$
83	2	not exists by [3, Theorem 2]
83	41	exists by [3, Example 1]
84	83	not exists by [3, Result 5]
85	2	not exists by [3, Theorem 2]
85	3	not exists by Theorem 4 (i) with $q=5$ and $u=1$
85	7	not exists by Theorem 4 (i) with $q=5$ and $u=1$
86	5	not exists by Theorem 4 (i) with $q=2$ and $u=1$
86	17	not exists by Theorem 4 (i) with $q=2$ and $u=1$
87	2	not exists by [3, Theorem 2]
87	43	not exists by Theorem 4 (i) with $q=3$ and $u=1$
88	3	not exists by Theorem 4 (i) with $q=2$ and $u=1$
88	29	not exists by Theorem 4 (i) with $q=2$ and $u=1$
89	2	not exists by [3, Theorem 2]
89	11	exists by [3, Example 1]
90	89	not exists by [3, Result 5]
91	2	not exists by [3, Theorem 2]
91	3	not exists by Theorem 5
91	5	not exists by Theorem 4 (i) with $q=7$ and $u=1$
92	7	not exists by Theorem 5
92	13	not exists by Theorem 4 (i) with $q=23$ and $u=1$
93	2	not exists by [3, Theorem 2]
93	23	not exists by Theorem 5
94	3	not exists by Theorem 4 (i) with $q=2$ and $u=1$
94	31	not exists by [9]
95	2	not exists by [3, Theorem 2]
95	47	not exists by Theorem 4 (i) with $q=5$ and $u=1$
96	5	not exists by Theorem 4 (i) with $q=2$ and $u=1$
96	19	not exists by Theorem 4 (i) with $q=2$ and $u=1$
97	2	not exists by [3, Theorem 2]
97	3	exists by [3, Example 1]
98	97	not exists by [3, Result 5]
99	2	not exists by [3, Theorem 2]
99	7	not exists by [9]
100	3	not exists by [9]
100	11	not exists by [9]

Table 7: almost p -ary nearly perfect sequences of period $n+1$ and type $\gamma = -1$.

n	p	Comments
2	2	exists by [3, Example 3]
3	3	not exists by [3, Theorem 7]
4	2	exists by [3, Example 3]
5	5	not exists by [3, Theorem 7]
6	2	exists by [3, Example 3]
7	3	exists by [3, Example 3]
8	2	not exists by Theorem 4 (ii) with $q=3$ and $u=9$
9	3	not exists by Theorem 4 (ii) with $q=5$ and $u=2$
10	2	exists by [3, Example 3]
11	5	exists by [3, Example 3]
12	11	not exists by [3, Theorem 7]
13	2	exists by [3, Example 3]
14	3	exists by [3, Example 3]
15	13	not exists by [3, Theorem 7]
16	2	not exists by Theorem 4 (ii) with $q=3$ and $u=5$
17	7	not exists by Theorem 4 (ii) with $q=5$ and $u=3$
18	3	not exists by Theorem 4 (ii) with $q=2$ and $u=16$
19	5	not exists by Theorem 4 (ii) with $q=2$ and $u=16$
20	2	exists by [3, Example 3]
21	17	not exists by [3, Theorem 7]
22	2	exists by [3, Example 3]
23	18	not exists by [3, Theorem 7]
24	3	exists by [3, Example 3]
25	3	exists by [3, Example 3]
26	19	not exists by [3, Theorem 7]
27	2	not exists by Theorem 4 (ii) with $q=3$ and $u=7$
28	5	
29	3	not exists by Theorem 4 (ii) with $q=2$ and $u=11$
30	7	
31	2	exists by [3, Example 3]
32	11	exists by [3, Example 3]
33	23	not exists by [3, Theorem 7]
34	2	not exists by Theorem 4 (ii) with $q=5$ and $u=25$
35	3	not exists by Theorem 4 (ii) with $q=5$ and $u=25$
36	5	not exists by Theorem 4 (ii) with $q=2$ and $u=13$
37	2	not exists by Theorem 4 (ii) with $q=3$ and $u=27$
38	13	
39	27	
40	2	exists by [3, Example 3]
41	7	exists by [3, Example 3]
42	29	not exists by [3, Theorem 7]
43	30	exists by [3, Example 3]
44	3	exists by [3, Example 3]
45	5	exists by [3, Example 3]
46	31	not exists by [3, Theorem 7]
47	32	not exists by Theorem 4 (ii) with $q=11$ and $u=3$
48	33	not exists by Theorem 4 (ii) with $q=17$ and $u=2$
49	11	not exists by Theorem 4 (ii) with $q=17$ and $u=2$
50	34	not exists by Theorem 4 (ii) with $q=5$ and $u=7$
51	17	
52	35	
53	7	not exists by Theorem 4 (ii) with $q=3$ and $u=36$
54	2	exists by [3, Example 3]
55	3	exists by [3, Example 3]
56	5	exists by [3, Example 3]
57	37	not exists by [3, Theorem 7]
58	38	
59	19	
60	3	not exists by Theorem 4 (ii) with $q=5$ and $u=2$
61	13	not exists by Theorem 4 (ii) with $q=2$ and $u=5$
62	2	exists by [3, Example 3]
63	40	exists by [3, Example 3]
64	5	exists by [3, Example 3]
65	41	not exists by [3, Theorem 7]
66	42	exists by [3, Example 3]
67	3	exists by [3, Example 3]
68	7	exists by [3, Example 3]
69	43	not exists by [3, Theorem 7]
70	44	not exists by Theorem 4 (ii) with $q=3$ and $u=45$
71	11	not exists by Theorem 4 (ii) with $q=17$ and $u=2$
72	3	not exists by Theorem 4 (ii) with $q=5$ and $u=2$
73	82	exists by [3, Example 3]
74	41	exists by [3, Example 3]
75	83	not exists by [3, Theorem 7]
76	84	not exists by Theorem 4 (ii) with $q=5$ and $u=17$
77	3	
78	7	
79	85	not exists by Theorem 4 (ii) with $q=43$ and $u=2$
80	17	not exists by Theorem 4 (ii) with $q=43$ and $u=2$
81	86	not exists by Theorem 4 (ii) with $q=3$ and $u=29$
82	43	not exists by Theorem 4 (ii) with $q=29$ and $u=3$
83	87	not exists by Theorem 4 (ii) with $q=2$ and $u=11$
84	29	not exists by Theorem 4 (ii) with $q=11$ and $u=2$
85	88	exists by [3, Example 3]
86	11	exists by [3, Example 3]
87	89	not exists by [3, Theorem 7]
88	90	not exists by Theorem 4 (ii) with $q=7$ and $u=13$
89	3	
90	5	not exists by Theorem 4 (ii) with $q=7$ and $u=13$
91	7	
92	13	not exists by Theorem 4 (ii) with $q=23$ and $u=2$
93	2	not exists by Theorem 4 (ii) with $q=3$ and $u=31$
94	23	
95	93	not exists by Theorem 4 (ii) with $q=47$ and $u=2$
96	31	
97	94	not exists by Theorem 4 (ii) with $q=19$ and $u=5$
98	47	not exists by Theorem 4 (ii) with $q=19$ and $u=5$
99	95	not exists by Theorem 4 (ii) with $q=3$ and $u=2$
100	19	not exists by Theorem 4 (ii) with $q=2$ and $u=3$
101	96	exists by [3, Example 3]
102	3	exists by [3, Example 3]
103	97	not exists by [3, Theorem 7]
104	98	not exists by Theorem 4 (ii) with $q=11$ and $u=3$
105	7	
106	99	not exists by Theorem 4 (ii) with $q=5$ and $u=50$
107	11	
108	100	exists by [3, Example 3]
109	5	exists by [3, Example 3]

11	not exists by Theorem 4 (ii) with $q=7$ and $u=2$
56	2 not exists by Theorem 4 (ii) with $q=3$ and $u=19$
7	not exists by Theorem 4 (ii) with $q=3$ and $u=19$
57	3 not exists by Theorem 4 (ii) with $q=29$ and $u=2$
19	not exists by Theorem 4 (ii) with $q=29$ and $u=2$
58	2 exists by [3, Example 3]
29	exists by [3, Example 3]
59	not exists by [3, Theorem 7]
60	2 exists by [3, Example 3]
3	exists by [3, Example 3]
5	exists by [3, Example 3]
61	not exists by [3, Theorem 7]
62	2 not exists by Theorem 4 (ii) with $q=3$ and $u=63$
31	not exists by Theorem 4 (ii) with $q=3$ and $u=63$
63	3 not exists by Theorem 4 (ii) with $q=2$ and $u=3$
7	
64	2 not exists by Theorem 4 (ii) with $q=5$ and $u=13$
65	5 not exists by Theorem 4 (ii) with $q=3$ and $u=2$
13	not exists by Theorem 4 (ii) with $q=11$ and $u=2$
66	2 exists by [3, Example 3]
3	exists by [3, Example 3]
11	exists by [3, Example 3]
67	not exists by [3, Theorem 7]
68	2 not exists by Theorem 4 (ii) with $q=23$ and $u=3$
17	
69	3 not exists by Theorem 4 (ii) with $q=5$ and $u=2$
23	not exists by Theorem 4 (ii) with $q=5$ and $u=2$
70	2 exists by [3, Example 3]
5	exists by [3, Example 3]
7	exists by [3, Example 3]
71	not exists by [3, Theorem 7]
72	2 exists by [3, Example 3]
3	exists by [3, Example 3]
73	not exists by [3, Theorem 7]
74	2 not exists by Theorem 4 (ii) with $q=3$ and $u=5$
37	
75	
5	not exists by Theorem 4 (ii) with $q=19$ and $u=2$
76	2 not exists by Theorem 4 (ii) with $q=7$ and $u=11$
19	
77	7 not exists by Theorem 4 (ii) with $q=3$ and $u=2$
11	not exists by Theorem 4 (ii) with $q=2$ and $u=3$
78	2 exists by [3, Example 3]
3	exists by [3, Example 3]
13	exists by [3, Example 3]
79	not exists by [3, Theorem 7]
80	2 not exists by Theorem 4 (ii) with $q=3$ and $u=27$
5	not exists by Theorem 4 (ii) with $q=3$ and $u=27$
81	3 not exists by Theorem 4 (ii) with $q=41$ and $u=2$
82	2 exists by [3, Example 3]
41	exists by [3, Example 3]
83	not exists by [3, Theorem 7]
84	2 not exists by Theorem 4 (ii) with $q=5$ and $u=17$
3	
7	
85	5 not exists by Theorem 4 (ii) with $q=43$ and $u=2$
17	not exists by Theorem 4 (ii) with $q=43$ and $u=2$
86	2 not exists by Theorem 4 (ii) with $q=3$ and $u=29$
43	not exists by Theorem 4 (ii) with $q=29$ and $u=3$
87	3 not exists by Theorem 4 (ii) with $q=2$ and $u=11$
29	not exists by Theorem 4 (ii) with $q=11$ and $u=2$
88	2 exists by [3, Example 3]
11	exists by [3, Example 3]
89	not exists by [3, Theorem 7]
90	2 not exists by Theorem 4 (ii) with $q=7$ and $u=13$
3	
5	not exists by Theorem 4 (ii) with $q=7$ and $u=13$
91	7
13	not exists by Theorem 4 (ii) with $q=23$ and $u=2$
92	2 not exists by Theorem 4 (ii) with $q=3$ and $u=31$
23	
93	3 not exists by Theorem 4 (ii) with $q=47$ and $u=2$
31	
94	2 not exists by Theorem 4 (ii) with $q=19$ and $u=5$
47	not exists by Theorem 4 (ii) with $q=19$ and $u=5$
95	5 not exists by Theorem 4 (ii) with $q=3$ and $u=2$
19	not exists by Theorem 4 (ii) with $q=2$ and $u=3$
96	2 exists by [3, Example 3]
3	exists by [3, Example 3]
97	not exists by [3, Theorem 7]
98	2 not exists by Theorem 4 (ii) with $q=11$ and $u=3$
7	
99	3 not exists by Theorem 4 (ii) with $q=5$ and $u=50$
11	
100	2 exists by [3, Example 3]
5	exists by [3, Example 3]

Table 8: almost p -ary nearly perfect sequences of period $n + 1$ and type $\gamma = 1$.

n	p	Comments
4	2	not exists by Theorem 4 (iii) resp. with $q=3$ and $u=5$
5	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
6	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
7	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
8	2	not exists by an exhaustive search
9	3	not exists by an exhaustive search
9	7	not exists by an exhaustive search
10	2	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
11	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
12	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
12	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
13	11	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
14	2	not exists by Theorem 4 (iii) with $q=7$ and $u=1$
14	3	not exists by an exhaustive search
15	13	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
16	2	not exists by Theorem 4 (iii) resp. with $q=3$ and $u=17$
17	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
17	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
18	2	not exists by an exhaustive search
19	17	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
20	2	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
20	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
21	19	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
22	2	not exists by Theorem 4 (iii) with $q=11$ and $u=1$
23	5	
23	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
24	7	
24	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
25	11	
25	23	
26	2	not exists by Theorem 4 (iii) with $q=13$ and $u=1$
27	3	
27	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
28	2	not exists by Theorem 4 (iii) with $q=7$ and $u=1$
29	13	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
29	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
30	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
30	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
31	29	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
32	2	not exists by an exhaustive search
33	3	
33	5	
33	31	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
34	2	not exists by Theorem 4 (iii) with $q=17$ and $u=1$
35	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
36	11	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
36	2	not exists by Theorem 4 (iii) resp. with $q=5$ and $u=37$
37	17	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
37	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
38	7	
38	2	not exists by Theorem 4 (iii) with $q=19$ and $u=1$
39	3	
39	37	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
40	2	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
41	19	
41	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
42	13	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
42	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
43	5	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
43	41	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
44	2	not exists by Theorem 4 (iii) with $q=11$ and $u=1$
44	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
45	7	
45	43	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
46	2	not exists by Theorem 4 (iii) with $q=23$ and $u=1$
47	11	
47	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
48	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
48	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
49	23	
49	47	
50	2	
50	3	
51	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
52	2	not exists by Theorem 4 (iii) with $q=13$ and $u=1$
53	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
53	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
54	17	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
54	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
55	13	
55	53	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
56	2	not exists by Theorem 4 (iii) with $q=7$ and $u=1$
57	3	not exists by Theorem 4 (iii) resp. with $q=5$ and $u=3$
57	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
58	11	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
58	2	not exists by Theorem 4 (iii) with $q=29$ and $u=1$
59	7	
59	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
59	19	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
60	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
60	29	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
61	59	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
62	2	not exists by Theorem 4 (iii) with $q=31$ and $u=1$
63	3	
63	5	
63	61	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
64	2	not exists by Theorem 4 (iii) resp. with $q=7$ and $u=5$
65	31	
65	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
66	7	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
66	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
67	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
67	13	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
68	2	not exists by Theorem 4 (iii) with $q=17$ and $u=1$
68	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
69	11	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
69	67	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
70	2	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
70	17	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
71	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
71	23	not exists by Theorem 4 (iii) resp. with $q=5$ and $u=2$
72	2	not exists by Theorem 4 (iii) resp. with $q=71$ and $u=73$
73	5	
73	7	
73	71	
74	2	not exists by Theorem 4 (iii) with $q=37$ and $u=1$
75	3	
75	73	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
76	2	not exists by Theorem 4 (iii) with $q=19$ and $u=1$
77	37	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
77	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
78	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
78	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
79	19	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
79	7	not exists by Theorem 4 (iii) resp. with $q=3$ and $u=2$
80	11	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
80	2	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
81	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
81	13	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
82	79	
82	2	not exists by Theorem 4 (iii) with $q=41$ and $u=1$
83	5	
83	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
84	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
84	41	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
85	83	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
86	2	not exists by Theorem 4 (iii) with $q=43$ and $u=1$
86	3	not exists by Theorem 4 (iii) resp. with $q=5$ and $u=3$
87	7	not exists by Theorem 4 (iii) resp. with $q=5$ and $u=3$
87	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
88	17	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
88	2	not exists by Theorem 4 (iii) with $q=11$ and $u=1$
89	43	
89	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
90	29	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
90	2	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
91	11	
91	89	not exists by Theorem 4 (iii) with $q=7$ and $u=1$
92	2	not exists by Theorem 4 (iii) with $q=23$ and $u=1$
93	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
93	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
93	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
94	13	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
94	2	not exists by Theorem 4 (iii) with $q=47$ and $u=1$
95	23	
95	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
96	31	
96	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
97	47	
97	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
98	19	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
98	2	
99	3	
99	97	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
100	2	not exists by Theorem 4 (iii) resp. with $q=11$ and $u=101$
101	7	

Table 9: almost p -ary nearly perfect sequences of period $n + 1$ and type $\gamma = 2$.

n	p	Comments
5	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
6	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
7	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
8	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
9	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	3	not exists by an exhaustive search
10	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
11	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
12	3	exists and given in Section 4
13	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	5	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
14	11	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
15	2	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
	3	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
16	13	
17	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
18	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
19	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
20	17	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
21	2	not exists by Theorem 4 (iii) with $q=7$ and $u=1$
	3	not exists by an exhaustive search
22	19	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
23	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	5	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
24	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	7	
25	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	11	
26	23	
27	2	not exists by an exhaustive search
	3	
28	5	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
29	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	13	
30	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
31	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
32	29	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
33	2	not exists by Theorem 4 (iii) with $q=11$ and $u=1$
	3	not exists by Theorem 4 (iii) with $q=11$ and $u=1$
	5	
34	31	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
35	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
36	3	
	11	
37	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	17	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
38	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
39	2	not exists by Theorem 4 (iii) with $q=13$ and $u=1$
	3	
40	37	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
41	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	19	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
42	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	13	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
43	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	5	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
44	41	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
45	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	3	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
46	43	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
47	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	11	
48	3	
	5	
49	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	23	
50	47	
51	2	not exists by Theorem 4 (iii) with $q=17$ and $u=1$
	3	not exists by Theorem 4 (iii) with $q=17$ and $u=1$
52	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
53	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	5	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
54	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	17	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
55	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	13	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
56	53	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
57	2	not exists by Theorem 4 (iii) with $q=19$ and $u=1$
	3	not exists by Theorem 4 (iii) resp. with $q=5$ and $u=2$
58	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	11	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
59	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
60	5	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
	19	
61	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	29	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
62	59	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
63	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	3	
	5	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
64	61	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
65	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	31	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
66	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	7	
67	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
68	5	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	13	not exists by Theorem 4 (iii) with $q=17$ and $u=1$
69	2	not exists by Theorem 4 (iii) with $q=23$ and $u=1$
	3	not exists by Theorem 4 (iii) with $q=23$ and $u=1$
	11	
70	67	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
71	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	17	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
72	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	23	
73	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	5	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
74	71	
75	2	
	3	
76	73	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
77	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	37	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
78	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
79	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	19	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
80	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	11	not exists by Theorem 4 (iii) resp. with $q=2$ and $u=3$
81	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	3	
	13	
82	79	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
83	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	5	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
84	3	
85	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	41	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
86	83	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
87	2	not exists by Theorem 4 (iii) with $q=29$ and $u=1$
	3	not exists by Theorem 4 (iii) with $q=29$ and $u=1$
	7	not exists by Theorem 4 (iii) resp. with $q=5$ and $u=2$
88	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	17	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
89	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	43	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
90	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	29	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
91	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	11	not exists by Theorem 4 (iii) with $q=7$ and $u=1$
92	89	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
93	2	not exists by Theorem 4 (iii) with $q=31$ and $u=1$
	3	
	5	not exists by Theorem 4 (iii) resp. with $q=7$ and $u=2$
94	7	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	13	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
95	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	23	not exists by Theorem 4 (iii) with $q=5$ and $u=1$
96	3	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	31	
97	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	47	not exists by Theorem 4 (iii) resp. with $q=5$ and $u=2$
98	5	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
	19	not exists by Theorem 4 (iii) with $q=2$ and $u=1$
99	2	not exists by Theorem 4 (iii) with $q=3$ and $u=1$
	3	not exists by Theorem 4 (iii) with $q=11$ and $u=1$
100	97	not exists by Theorem 4 (iii) with $q=3$ and $u=1$